ONLINE NMPC OF A LOOPING KITE USING APPROXIMATE INFINITE HORIZON CLOSED LOOP COSTING

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Abstract: We consider a dual line kite that shall fly loops and address this periodic control problem with a state-of-the-art NMPC scheme. The kite is described by a nonlinear unstable ODE system, and the aim is to let the kite fly a periodic figure. Our approach is based on the 'infinite horizon closed loop costing" NMPC scheme to ensure nominal stability. To be able to apply this scheme, we first determine a periodic LQR controller to stabilize the kite locally in the periodic orbit. Then, we formulate a two-stage NMPC optimal control problem penalizing deviations of the system state from the periodic orbit, which also contains a state constraint that avoids that the kite collides with the ground. To solve the optimal control problems reliably and in real-time, we apply the newly developed "real-time iteration scheme" for fast online optimization in NMPC. The optimization based NMPC leads to significantly improved performance compared to the LQR controller, in particular as it respects state constraints. The NMPC closed loop also shows considerable robustness against changes in the wind direction.

Keywords: periodic control, LQR, nonlinear control systems, predictive control, online optimization, stability, numerical methods, optimal control

1. INTRODUCTION

Nonlinear model predictive control (NMPC) is a feedback control technique that is based on the real-time optimization of a nonlinear dynamic process model on a moving horizon that has attracted increasing attention over the past decade (Qin and Badgwell, 2001). Important challenges that need to be addressed for any NMPC application are stability of the closed loop system and the numerical solution of the optimal control problems in real-time. In this paper we show how state-of-the-art NMPC techniques addressing these challenges can be applied to control a strongly unstable periodic system, namely a dual line kite that shall fly loops. The aim of our automatic control is to make the kite fly a figure that may be called a "lying eight". The corresponding orbit is not open loop stable, so that feedback has to be applied. We assume the state is fully accessible for control.

Since the natural setting of the problem is in continuous time, the NMPC implementation proposed here is developed for continuous-time systems. However, it basically differs from the continuous time NMPC algorithms for nonlinear systems previously published in the literature, see e.g. (Mayne and Michalska, 1990; Chen and Allgöwer, 1998). Continuous time methods usually assume that the NMPC law is continuously computed by solving at any time instant a difficult optimization problem. This is impossible in practice, as any implementation is performed in digital form and requires a non-negligible computational time. The NMPC setup proposed here is based on the method proposed in (Magni *et al.*, 2002), where a continuous

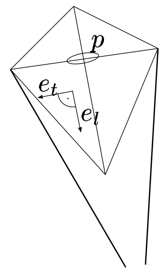

Fig. 1. A picture of the kite, and the unit vectors e_l and e_t along longitudinal and transversal kite axis.

time locally stabilizing control law is first designed. Then, a piecewise constant term computed via NMPC is added to the control signal provided by the stabilizing control law, in order to achieve some specific goals, such as the minimization of a prescribed cost or the enlargment of the output admissible set. In so doing, it is assumed that the signal computed by NMPC is piecewise constant and with a limited number of free moves in the future. Nominal stability of the overall system is preserved using the "infinite horizon closed loop costing" scheme proposed in (De Nicolao *et al.*, 1998). In the usual setting of this scheme, the optimization problems are solved up to a prespecified accuracy during each sampling time so that a feedback delay of one sampling time is introduced in the closed loop.

In this paper, however, we avoid this feedback delay by using the recently developed "real-time iteration" scheme (Diehl *et al.*, 2002*b*) for online optimization. The algorithm is based on the direct multiple shooting approach to optimal control problems (Bock and Plitt, 1984; Leineweber, 1999), but is characterized by the following features: first, the scheme efficiently initializes each new problem and performs only one optimization iteration per optimization problem. Thus it reduces sampling times to a minimum. Second, the computations of each "real-time iteration" are divided into a very short "feedback phase", and a much longer "preparation phase", which uses the sampling time to *prepare* the next feedback. Thus, each NMPC feedback is directly applied to the system, with a negligible delay that is orders of magnitude shorter than the sampling time. For details see e.g. (Diehl *et al.*, 2002*c*).

The paper is organized as follows. In Section 2 we derive the model equations for the kite model. The periodic reference orbit is analysed in Section 3 and we show how to design a stabilizing periodic linear controller based on LQR techniques. In Section 4 we finally describe the NMPC setup. Simulated closed loop experiments are presented and briefly discussed in Section 5.

2. KITE MODEL

The kite is held by two lines which allow to control the lateral angle of the kite, see Fig. 1. By pulling one line the kite will turn in the direction of the line being pulled. In this paper we employ a kite model that was originally developed in (Diehl, 2002; Diehl *et al.*, 2002*a*).

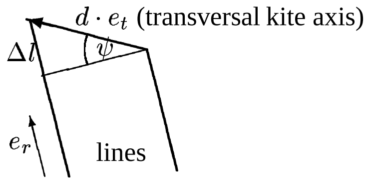

Fig. 2. The kite and its lines seen from the top, and visualization of the lateral angle ψ.

2.1 Kite Dynamics in Polar Coordinates

The movement of the kite at the sky can be modelled by Newton's law of motion and a suitable model for the aerodynamic force. Let us introduce polar coordinates θ, ϕ, r so that the position p of the kite relative to the kite pilot (in the origin) is given by: $p = (r\sin(\theta)\cos(\phi), r\sin(\theta)\sin(\phi), r\cos(\theta))$ with the last component being the height of the kite over the ground, and θ being the angle that the kite lines form with the vertical. We introduce a local right handed coordinate system with the basis vectors e_θ, e_ϕ, e_r, each pointing in the direction where the corresponding polar coordinate increases. Defining the corresponding components of the total force F acting on the kite, we can write Newton's law of motion for constant r in the form

$$\ddot{\theta} = \frac{F_\theta}{rm} + \sin(\theta)\cos(\theta)\dot{\phi}^2, \tag{1}$$

$$\ddot{\phi} = \frac{F_\phi}{rm\sin(\theta)} - 2\cot(\theta)\dot{\phi}\dot{\theta}, \tag{2}$$

with m denoting the mass of the kite. The force consists of two contributions, gravitational and aerodynamic force, so that we obtain $F_\theta = \sin(\theta)mg + F_\theta^{\text{aer}}$ and $F_\phi = F_\phi^{\text{aer}}$, where g = 9.81 m s^{-2} is the earth's gravitational acceleration. It remains to determine the aerodynamic forces F_ϕ^{aer} and F_θ^{aer}.

2.2 Kite Orientation

To model the aerodynamic force we first determine the kite's orientation. We assume that the kite's trailing edge is strongly pulled by the tail into the direction of the effective wind at the kite. Under this assumption the kite's longitudinal axis is always in line with the effective wind vector $w_e := w - \dot{p}$, where $w = (v_w, 0, 0)^T$ is the wind as seen from the earth system, and $\dot{p}$ the kite velocity. If we introduce a unit vector e_l pointing from the front towards the trailing edge of the kite (cf. Fig. 1), we therefore assume that $e_l = \frac{w_e}{\|w_e\|}$. The transversal axis of the kite can be described by a perpendicular unit vector e_t that is pointing from the right to the left wing tip (as seen from the kite pilot, in upright kite orientation). The orientation of e_t can be controlled, but it has to be orthogonal to e_l (cf. Fig. 1),

$$e_t \cdot e_l = 0. \tag{3}$$

However, the projection of e_t onto the lines' axis (which is given by the vector e_r) is determined from the length difference Δl of the two lines, see Fig. 2. If the distance between the two lines' fixing points on the kite is d, then the vector from the right to the left fixing point is $d \cdot e_t$, and the projection of this vector onto the lines' axis should equal Δl (being positive if the left wing tip is farther away from the pilot), i.e., $\Delta l = d\, e_t \cdot e_r$. Let us define the *lateral angle* ψ to be $\psi = \arcsin\left(\frac{\Delta l}{d}\right)$. For simplicity, we assume that we control this angle ψ directly. It determines the orientation of e_t which has to satisfy:

$$e_t \cdot e_r = \frac{\Delta l}{d} = \sin(\psi). \qquad (4)$$

A third requirement that e_t must satisfy and which determines its sign is

$$(e_l \times e_t) \cdot e_r > 0. \qquad (5)$$

This ensures that the kite is always in the same orientation with respect to the lines. We will now give an explicit form for the unit vector e_t that satisfies (3)–(5). Using the projection $\bar{w}_e$ of the effective wind vector w_e onto the tangent plane spanned by e_θ and e_ϕ, $\bar{w}_e := w_e - e_r(e_r \cdot w_e)$, we can define the orthogonal unit vectors $e_1 := \frac{\bar{w}_e}{\|\bar{w}_e\|}$ and $e_2 := e_r \times e_1$, so that (e_1, e_2, e_r) forms an orthogonal right-handed coordinate basis. In this basis the effective wind w_e has no component in the e_2 direction: $w_e = \|\bar{w}_e\| e_1 + (w_e \cdot e_r) e_r$. The definition

$$e_t := e_1(-\cos\psi \sin\eta) + e_2(\cos\psi\cos\eta) + e_r \sin\psi$$

with $\eta := \arcsin\left(\frac{w_e \cdot e_r}{\|\bar{w}_e\|}\tan(\psi)\right)$ indeed satisfies (3)–(5), as can be verified by direct substitution. Therefore we are now able to determine the orientation of the kite depending on the control ψ and the effective wind w_e only.

2.3 Aerodynamic Lift and Drag

The two vectors $e_l \times e_t$ and e_l are the directions of aerodynamic lift and drag, respectively. To compute the magnitudes F_L and F_D of lift and drag we assume that the lift and drag coefficients C_L and C_D are constant, so that we have

$$F_\mathrm{L} = \frac{1}{2}\rho\|w_e\|^2 A C_\mathrm{L} \quad \text{and} \quad F_\mathrm{D} = \frac{1}{2}\rho\|w_e\|^2 A C_\mathrm{D},$$

with ρ being the density of air, and A being the characteristic area of the kite. Given the directions and magnitudes of lift and drag, we can compute F^aer as their sum, yielding $F^\mathrm{aer} = F_\mathrm{L}(e_l \times e_t) + F_\mathrm{D} e_l$ or, in the local coordinate system

$$F_\theta^\mathrm{aer} = F_\mathrm{L}((e_l \times e_t)\cdot e_\theta) + F_\mathrm{D}(e_l \cdot e_\theta)$$
$$F_\phi^\mathrm{aer} = F_\mathrm{L}((e_l \times e_t)\cdot e_\phi) + F_\mathrm{D}(e_l \cdot e_\phi).$$

The system parameters that have been chosen for the simulation model are listed e.g. in (Diehl, 2002; Diehl *et al.*, 2002*a*). Defining the system state $x := (\theta, \phi, \dot\theta, \dot\phi)^T$ and the control $u := \psi$ we can summarize the four system equations, i.e., (1)–(2) and the trivial equations $\frac{\partial\theta}{\partial t} = \dot\theta$, $\frac{\partial\phi}{\partial t} = \dot\phi$, in the short form $\dot{x} = f(x, u)$.

3. REFERENCE ORBIT AND LQR

Using the above system model, a periodic orbit was determined that can be characterized as a "lying eight" and which can be seen e.g. in Figure 3, as a (ϕ, θ)-plot. The wind is assumed to blow in the direction of the p_1-axis ($\theta = 90^o$ and $\phi = 0^o$). The periodic solution was computed using an off-line variant of the direct multiple shooting method, MUSCOD-II (Leineweber, 1999), imposing periodicity conditions with period $T := 8$ seconds and suitable state bounds and a suitable objective function in order to yield a solution that we considered to be a nice reference orbit. We denote the periodic reference solution by $x_r(t)$ and $u_r(t)$. This solution is defined for all $t \in (-\infty, \infty)$ and satisfies the periodicity condition $x_r(t+T) = x_r(t)$ and $u_r(t+T) = u_r(t)$.

3.1 Open Loop Stability Analysis

Numerical simulations of the kite using the open loop inputs $u_r(t)$ show that the kite crashes onto the ground very quickly after small disturbances, cf. Fig. 3. To analyse the asymptotic stability properties of the open loop system along the periodic reference orbit theoretically, let us consider the linearization of the system along the orbit. An infinitesimal deviation $\delta x(t) := x(t) - x_r(t)$ and $\delta u(t) := u(t) - u_r(t)$ would satisfy the periodically time-varying linear differential equation

$$\delta\dot{x}(t) = A(t)\delta x(t) + B(t)\delta u(t), \qquad (6)$$

with

$$A(t) := \frac{\partial f}{\partial x}(x_r(t), u_r(t)) \quad \text{and} \quad B(t) := \frac{\partial f}{\partial u}(\cdot).$$

Based on the linear time variant periodic system $\dot{x}(t) = A(t)x(t)$ we can compute its fundamental solution and the sensitivity of the final state of each period with respect to its initial value, which is called the "monodromy matrix". A numerical computation of the monodromy matrix for the kite orbit and eigenvalue decomposition yields two Eigenvalues ("Floquet multipliers") that are greater than one, confirming the observation that the system is unstable.

3.2 Design of a Periodic LQR Controller

In order to design a locally stabilizing controller (which is needed if we want to apply the infinite horizon closed loop costing NMPC scheme), we use the classical LQR design technique, applied to the periodic linear system (6). We introduce diagonal weighting matrices

$$Q := \mathrm{diag}(0.4,\ 1,\ \mathrm{s}^2,\ \mathrm{s}^2)\frac{1}{\mathrm{s}} \text{ and } R := 33\,\frac{1}{\mathrm{s}}. \qquad (7)$$

To determine the optimal periodic LQR controller that minimizes the objective $\int \frac{1}{2}(x - x_r(t))^T Q (x - x_r(t)) + \frac{1}{2}(u - u_r(t))^T R (u - u_r(t)) dt$, we find the symmetric periodic matrix solution $P(t)$ for the differential Riccati equation

$$-\dot{P} = Q + A(t)^T P(t) + P(t)A(t) - P(t)B(t)R^{-1}B(t)^T P(t)$$

by integrating the equation backwards for a sufficiently long time, starting with the unit matrix as final value. Once the periodic $P(t)$ is determined, the optimal LQR controller for (6) and (7) is given by $\delta u(t) = -K(t)\delta x(t)$ with $K(t) := R^{-1}B(t)^T P(t)$. We finally define the linear periodic feedback for the original system as

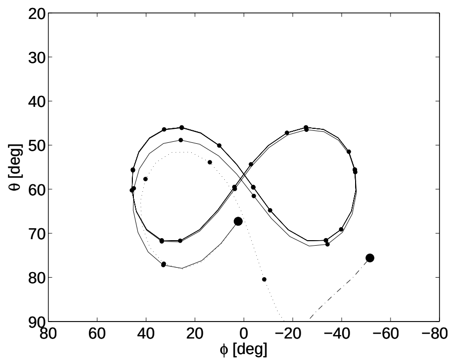

Fig. 3. The linear periodic controller is able to stabilize the system locally from slightly disturbed system states (solid line), in contrast to the open loop system (dotted), but lets the kite crash onto the ground for a larger deviation (dash dotted).

$$u_{\mathrm{LQR}}(x,t) := u_r(t) - K(t)(x - x_r(t)). \quad (8)$$

The linearly stabilized system $\dot{x}(t) = f(x(t), u_{\mathrm{LQR}}(x(t),t))$ is locally stable, as illustrated in Figure 3.

4. NMPC CONTROLLER SETUP

The aim of the NMPC controller is to stabilize the system in a larger region of attraction and to respect certain bounds that the linear controller may violate, and, furthermore, to lead to an improved performance with respect to a user defined criterion. In our case, the bounds arise because we want the closed loop kite to respect a security distance to the ground ($\theta = 90$ deg). We achieve this by requiring that $h(x,u,t) := 76.5\,\mathrm{deg} - \theta \geq 0$. The performance of the controller is measured by the integral of a function $L(x(t),u(t),t)$, which is in our case the squared deviation of the state x at time t from the reference orbit,

$$L(x,u,t) := \frac{1}{2}(x - x_r(t))^T Q (x - x_r(t)).$$

We introduce a sampling interval δ and give NMPC feedback to the system only at the times $t_k := k \cdot \delta$. At each sampling time t_k the NMPC shall deliver new controls $u(t), t \in [t_k, t_{k+1}]$ that depend on the current system value $x(t_k)$, where the optimization is based on a prediction of the future system behaviour. Many NMPC schemes exist that guarantee nominal stability, see e.g. (Allgöwer *et al.*, 1999; De Nicolao *et al.*, 2000); they mainly differ in the way the optimal control problems are formulated. Here, we work in the framework of the infinite horizon closed loop costing scheme (De Nicolao *et al.*, 1998).

4.1 Infinite Horizon Closed Loop Costing

In the infinite horizon closed loop costing scheme we express the control $u(t)$ that is actually applied to the plant at time $t \in [t_k, t_{k+1}]$ by the sum

$$u(t) = u_{\mathrm{LQR}}(x(t),t) + v_k \quad \forall t \in [t_k, t_{k+1}],$$

where the constant vector v_k is determined by the NMPC optimizer and implicitly depends on $x(t_k)$. Note that $v_k \equiv 0$ yields the linearly controlled closed loop. In the sequel we will use a bar to distinguish the predicted system state and controls $\bar{x}(t)$ and $\bar{u}(t)$ from the state and control vector of the real system.

Given the state $x(t_k)$ of the "real" kite at time t_k, we formulate the following optimal control problem, with control horizon $T_c = M\delta$ and prediction horizon $T_p > T_c$ (where T_p shall ideally be infinity).

$$\min_{\bar{v}_i, \bar{u}(\cdot), \bar{x}(\cdot)} \int_{t_k}^{t_k+T_p} L(\bar{x}(t), \bar{u}(t), t)\, dt \quad (9)$$

subject to

$$\begin{aligned} \dot{\bar{x}}(t) &= f(\bar{x}(t), \bar{u}(t)), \quad \forall t \in [t_k, t_k + T_p], \\ \bar{x}(t_k) &= x(t_k), \\ \bar{u}(t) &= u_{\mathrm{LQR}}(\bar{x}(t),t) + \bar{v}_i, \quad \forall t \in [t_i, t_{i+1}], \\ &\qquad (i = k, \ldots, k+M-1), \\ \bar{u}(t) &= u_{\mathrm{LQR}}(\bar{x}(t),t), \ \forall t \in [t_k + T_c, t_k + T_p] \\ 0 &\leq h(\bar{x}(t), \bar{u}(t), t), \ \forall t \in [t_k, t_k + T_p]. \quad (10) \end{aligned}$$

In the case that $T_p = \infty$ and if the optimal control problem has a solution for $x(t_0)$, stability of the closed loop trajectory can be proved in a rigorous way (De Nicolao *et al.*, 1998; Magni *et al.*, 2002).

4.2 Real-Time Optimization Scheme

We choose a sampling interval $\delta = 1$ s and $M = 8$ sampling intervals as control horizon, $T_c = 8$ s. As the simulation of the periodic system over an infinite horizon is impossible, we employ here a finite $T_p = 24$ s, that we believe to be sufficiently long to deliver a fair approximation to the infinite horizon cost. For a theoretical discussion on how to truncate the series expressing the Infinite Horizon cost associated with the auxiliary linear control law without losing stability see (Magni *et al.*, 2001). Furthermore, to avoid a semi-infinite optimization problem, the problem is changed by imposing the inequality path constraints (10) only at prespecified points in time, here chosen to be the sampling times t_i on the control horizon, as well as start, center and end point of the prediction horizon.

The numerical solution of the optimization problems is achieved by the recently developed real-time iteration scheme (Diehl *et al.*, 2002*b*; Diehl *et al.*, 2002*c*) that is based on ideas developed in (Bock *et al.*, 2000). This scheme is based on the direct multiple shooting method (Bock and Plitt, 1984) that reformulates the optimization problem as a finite dimensional nonlinear programming problem with a special structure. We use an efficient implementation of the scheme within the optimal control package MUSCOD-II (Leineweber, 1999). One advantage of the scheme is that it nearly completely avoids the feedback delay of one sampling time present in most NMPC optimization schemes.

5. CLOSED LOOP EXPERIMENTS

In order to test the NMPC closed loop we have performed several numerical experiments. Here, the "real" kite is simulated by a model that coincides with the optimization model, but is subject to disturbances of different type.

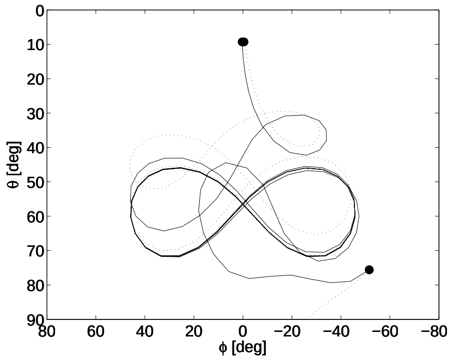

Fig. 4. The NMPC controller is able to control the kite even for the largely disturbed states at the bottom (solid line), in contrast to the LQR controller (dotted, cf. Fig. 3). For the disturbed state at the top, the performance for the NMPC (solid line, integrated costs 1.51) is better that that of the LQR (dotted, costs 1.75), as expected.

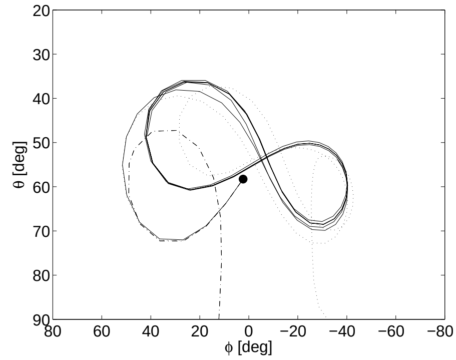

Fig. 5. Effect of model uncertainty in form of a strong side wind with 50 % the nominal wind speed. After a very short transient, the NMPC controlled kite loops in a considerably disturbed periodic orbit, but remains stable (solid line). The LQR closed loop response results in a crash after two periods (dotted), and the open loop crashes after 4 seconds (dash dotted).

5.1 Comparison with LQR

First, let us in Figure 4 compare the NMPC with the periodic LQR. It can be seen that the NMPC is able to respect the state constraint $\theta \leq 76.5$ deg even for the scenario with the largely disturbed initial state at the bottom (cf. Fig. 3), in contrast to the LQR. For another scenario, where the system kite starts much too high at the sky, both controllers are able to stabilize the system. However, NMPC leads to a reduced objective with the cost integral $\int_0^\infty L(x(t), u(t), t)dt$ being 1.51 in contrast to 1.75 for the LQR. This difference can be seen in form of a considerably faster convergence towards the periodic orbit.

5.2 Strong Sidewind

In another scenario, the closed loop is tested against model uncertainty: we consider a continuing disturbance resulting from a change in the wind direction. The wind component in p_2-direction, that is assumed by the optimizer to be zero, is for the "real" kite set to a value 3 m/s that is 50% of the nominal wind v_w in p_1 direction. The NMPC closed loop results in a considerably disturbed but stable periodic orbit, as can be seen in Figure 5; the disturbed periodic orbit is reached after a very short transient. This contrasts sharply with the open loop and the LQR closed loop response which both result in a crash after a short time.

6. CONCLUSIONS

We have presented a method to design a nonlinear model predictive controller for periodic unstable systems, and have applied the method to a kite that shall fly loops. The method is based on the "infinite horizon closed loop costing" which requires a locally prestabilizing feedback. This prestabilization is achieved by a periodic LQR controller based on a system linearization along the periodic orbit. The NMPC controller uses an objective which only penalizes state deviations and a state constraint is formulated to ensure that the kite does not crash onto the ground. The resulting optimal control problems are solved in real-time, once a second, by a state-of-the-art online optimization algorithm, the "real-time iteration scheme". This numerical scheme avoids the large feedback delay present in most optimization approaches to NMPC and allows to reduce sampling times to a minimum. The NMPC closed loop gives an excellent response to strong disturbances. Furthermore, it shows good robustness against model plant mismatch: in the presence of additional sidewind of 50% the nominal wind velocity the periodic orbit changes shape, but remains stable.

We want to mention here that the real-time iteration NMPC scheme used for the computations in this paper has also been successfully applied to a real pilot scale distillation column described by a stiff differential-algebraic equation model with 200 states, making a sampling time of 20 seconds possible (Diehl *et al.*, 2003).

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