Graphical Abstract

Techno-economic sensitivity analysis for combined design and operation of a small modular reactor hybrid energy system

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Highlights

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- Optimize SMR-renewable hybrid energy system design with uncertain parameters
- Quantify design changes with forecasts as stochastic time series
- Investigate optimal dispatch and design with varying horizon length
- Method for sensitivity analysis of combined design and dispatch of SMR hybrid energy systems
Techno-economic sensitivity analysis for combined design and operation of a small modular reactor hybrid energy system

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\textbf{Abstract}

With increasing grid-penetration of renewable energy resources and a rising need for carbon-free dispatchable power generation, nuclear-hybrid energy systems (NHES), consisting of small modular reactors, are an increasingly attractive option for maintaining grid stability. NHES can accomplish this with a minimal carbon footprint but there are significant uncertainties that are not fully understood. This work describes and demonstrates methods for analyzing the uncertainties of potential NHES designs, including uncertain design parameters and time series as well as variations in dispatch horizon length. The proposed methods are demonstrated on a sample system with 16 design parameters, 3 uncertain time series, and a range of dispatch horizon lengths where the unit capacities and unit dispatch are co-optimized to minimize system LCOE. For the example system, 11 of 16 parameters are uncorrelated with model outputs, allowing for model reduction without decreased accuracy. It is determined that the impact of variation in multiple time series cannot be easily isolated and that the examined sources of uncertainty are of similar importance in terms of overall impact.

\textit{Keywords:} hybrid energy system, design optimization, sensitivity analysis, uncertainty quantification

\textbf{Nomenclature}

NHES nuclear-hybrid energy system
LCOE levelized cost of electricity
SMR small modular reactor
TES thermal energy storage
NPP nuclear power plant
$\eta_{\text{turb}}$ steam turbine efficiency
1. Introduction

Availability of electrical power is a key performance index of a society [10]. Reliable electrical power is critical for sustainable and safe industrial settings and is a growing necessity for domestic and commercial settings [33]. The use of solar, wind, and renewable energy resources is increasing rapidly and expected to continue to do so [16] [33]. Huber et al. and Bertsch et al. highlight multiple cases in which increasing variable energy resource penetration requires increased flexibility in the electrical grid [25] [9]. Increasing numbers of electric vehicles also place unique demands on the grid [2] that may motivate more flexible generation. To address this need, notable works have proposed hybrid energy systems (HES) in which a reliable base load power source and energy storage meet any demands that exceed the stochastic supply produced by renewable energy sources [48] [21] [20]. Nuclear energy is a reliable and low emission base load, making nuclear-hybrid energy systems (NHES) a favorable option for maintaining grid stability in the future. New forms of thermal energy storage also have potential for reduced environmental impact while facilitating hybrid energy systems that provide greater flexibility [13].

Recent publications demonstrate various NHES designs that are unique combinations of energy generation technologies. Zhao et al. considers an NHES design combining small modular reactors (SMRs), concentrated solar power, and thermal energy storage (TES) [49].
Ho et al. combines SMRs with large-scale hydrogen storage as a means of increasing grid flexibility [24]. Other designs include industrial applications, such as Baker et al. with a desalination plant, Kim et al. with a high temperature steam electrolysis plant (HTSE), and Ozcan et al. with hydrogen production using the Mg-Cl cycle [7, 30, 38]. Abdusammi et al. and Wang et al. implement unique subsystems to increase cost efficiencies in conceptual NHES [1, 46]. These designs, among others, represent significant groundwork in the development of NHES.

Knowledge gaps surrounding system variability and dynamics prevent the advancement of economic and optimal NHES solutions. Unrealized designs come with an innate shortage of related data, complicating any manipulation of the system for specific conditions or demands. Successful large-scale application of NHES will require thorough groundwork to enable flexibility for regional needs. This groundwork includes determining the sensitivity of the design and dispatch parameters addressed in this paper. The impact of variation in system parameters is often non-intuitive and several publications focus on dealing with uncertainties inherent to NHES. Garcia et al. and Chen et al. eliminate many case specific uncertainties in their regional examples, but leave a more generalized approach for future research [20]. Abdusammi et al. study the sensitivity of several design parameters, but highlights the need for more complete work including uncertain parameters related to renewable energy resources and demand [1].

Renewable resource components present naturally challenging factors in the simulation of complex systems, with both unique patterns and uncertainties. Idaho National Laboratory (INL) provides several tools to handle the stochastic nature of NHES [11, 40, 18]. INL’s Risk Analysis Virtual ENvironment (RAVEN) produces synthetic uncertain time series at a large scale, enabling a more thorough analysis of untested designs [39]. Optimization of the dispatch and design are handled separately, providing useful information about stochastic component behavior. Additional system uncertainties that are not addressed include system parameters, costs, and dynamics.

Each of the works modeling NHES makes use of the idea of a dispatch horizon, or the time horizon over which the system is dispatched, in one form or another. For many works, this is one full year [24, 23], but others time lengths are used in literature as well [14] and there is no clear consensus on the best length to be used for the dispatch horizon.

Uncertainty and sensitivity analyses are thoroughly tested methods for developing a knowledge base of complex systems [32]. Sensitivity analysis in HES is well developed and is frequently used for case-specific research and optimization [6, 29, 4]. Levelized cost of electricity (LCOE) is commonly used as an economic performance metric for parameter analysis [42, 4]. Case-specific studies have great value, but a broader approach is needed to establish guidelines for new designs. Tian et al. and Stelt et al. examine fixed combinations of components in more generalized sensitivity analyses to identify which parameters carry the most weight [43, 41]. The economic feasibility of each system is determined and key parameters are highlighted.

There is no detailed sensitivity analysis for a general NHES. Lacking further insight, many system parameters are exhaustively accounted for without evidence of their relative...
influence or variation in these parameters is ignored entirely. Using principles from well-established disciplines, this paper presents sensitivity analysis methods tailored to typical NHES models and provides useful insight into how to best account for uncertainties in NHES. A unique combined dispatch and design nonlinear optimization model in GEKKO Python is supported by INL developed time series tools to fully account for system uncertainties. Interactions between design and dispatch parameters are quantified along with relative impact on economic feasibility. The results provide important insight for how to account for uncertainties for future NHES implementation.

The rest of this article is organized as follows: The sample NHES design is described and then formulated in Section 2. Implementation of the simulated model and sensitivity analysis methods are discussed in Section 3. Section 4 covers an analysis of the data and resulting conclusions are presented in Section 5.

2. Material and Methods

The NHES design analyzed in this work consists of a nuclear power plant (NPP) made up of small modular reactors (SMR), a photovoltaic solar field, a wind farm, and a thermal energy storage (TES) unit meeting a set electrical load. Each of the system components are considered fully-integrated with each other and transmission losses of electricity and steam resources are considered negligible. No other generators or energy storage devices are used to meet the load. The units are operated in a coordinated manner allowing them to minimize the overall levelized cost of electricity (LCOE). This means that the dispatch of each of the dispatchable components is manipulated by a single optimizer. A figure demonstrating the interconnections between the components is shown in Figure 1, where the thermal energy connections are represented in red and the electrical connections are represented in blue.

The NPP is modeled as a generic, large-scale steam producer with economic and dynamic operational parameters. The steam produced by the NPP is directed to the steam turbine, the TES unit, or both, depending on the system dispatch. The output of the plant is considered to be flexible over time but with imposed economic costs that encourage minimal power output manipulation. While the economic costs associated with load-following SMR NPPs are expected to be minimal or negligible, various other costs (legal, safety, control difficulties, etc.) could be included here as economic costs to appropriately drive the optimization. More complex aspects of nuclear power plants, such as fuel reloading and core neutronics, are not considered in this work.

The electricity generated from the wind farm and photovoltaic solar field combines with the electricity generated in the steam turbine to provide the generation capacity necessary to meet the system load. The contribution from renewable sources must be utilized by the system dispatcher, but cannot be manipulated or curtailed in this study.

The system is required to provide the electrical load while utilizing both the dispatchable and non-dispatchable resources, but overproduction is allowed. Any overproduction in the system is penalized in the optimization but this penalty is not included in the LCOE. NHES systems are expected to be able to handle overproduction through renewable generation curtailment or by allowing excess steam to bypass the steam turbines although over-generation
is not optimal [19].

The TES unit is a two-tank design consisting of insulated cold and hot storage tanks filled with a molten salt. Steam charging the TES heats salt from the cold tank after which it is stored in the hot tank. When discharging, the salt from the hot tank is used to heat process water or steam before sending it to the steam turbine. A roundtrip efficiency is used to approximate the efficiency of the TES.

The sizes of the NPP, TES unit, and steam turbine as well as the operation of these units are not fixed, but are manipulated during model optimization. The system load, wind farm capacity, and solar farm capacity are fixed in this case study scenario.

2.1. Mathematical Description

The model can be described mathematically as shown in Equation 1.
minimize \( LCOE + \frac{0.1}{n_{hrs}} (E_{turb,in} \eta_{turb} + E_{solar} + E_{wind} - E_{load}) \) \quad (1a)

subject to \( LCOE = \frac{\sum_{i}^{n} (C_{cap,i} N_i L_{sys} L_i + C_{foc,i} L_{sys} + C_{voc,i} \int_{0}^{n_{hrs}} E_{i,t} dt) + D_{cost}}{\int_{0}^{n_{hrs}} E_{load,t} dt} \) \quad (1b)

\( E_{smr} - E_{tes,in} + E_{tes,out} \eta_{tes} = E_{turb,in} \) \quad (1c)

\( E_{turb,in} \eta_{turb} + E_{sol} + E_{wind} = E_{load} \) \quad (1d)

\( N_{min,smr} N_{smr} \leq E_{smr} \leq N_{smr} \) \quad (1e)

\( 0 \leq E_{turb,in} \eta_{turb} \leq N_{turb} \) \quad (1f)

\( r_{down} \leq \frac{dE_{smr}}{dt} \leq r_{up} \) \quad (1g)

\( 0 \leq S \leq N_{tes} \) \quad (1h)

\( D_{cost} = D_{cost,smr} \int_{0}^{n_{hrs}} \left( \frac{dE_{smr,t}}{dt} \right)^2 + D_{cost,tes} \int_{0}^{n_{hrs}} (E_{tes,in,t} + E_{tes,out,t}) dt \) \quad (1i)

2.1.1. Objective Function

The objective function, as shown in Equation 1a, is to minimize the sum of the system LCOE over the dispatch horizon length while penalizing overproduction of electricity. The calculation of the LCOE itself is formulated as a constraint in the optimization, as shown in Equation 1b. The second term in the objective function allows penalizing overproduction without that penalty contributing to the LCOE. It is weighted light enough to discourage overproduction without driving the optimization to economically unfavorable dispatch profiles.

2.2. Decision Variables

The decision variables for the optimization problem consist of design and dispatch variables. The design variables \( x_d \) for this problem are the capacities of the SMR \( (N_{SMR}) \), turbine \( (N_{turb}) \), and TES \( (N_{TES}) \). The dispatch variables \( x_{op} \) for the problem the amount of energy produced by the SMR at each point in time \( (E_{SMR}) \) and the amounts of energy stored \( (E_{TES,in}) \) or retrieved \( (E_{TES,out}) \) from the TES at each point in time. The design parameters, time series, and dispatch horizon length are taken as model inputs, but cannot be modified by the optimizer. Both the design and dispatch variables are simultaneously co-optimized.

2.3. Constraints

The constraints on the optimization problem are given in Equations 1b-1i. These constraints ensure the feasibility of the solution by ensuring that the heat and electricity balances are satisfied, that the units are operating within the respective constraints for those units and that the LCOE is calculated accurately.
The LCOE, shown in Equation 1b, is formulated as a sum of the costs from each component divided by the total power output of the system. Capital costs of \( n \) components are scaled with regard to system lifetime to account for continued value beyond system exhaustion. Fixed operational costs refer to costs from standard operation and maintenance. Variable operational costs are costs related to fluctuations in component use. The \( D_{\text{cost}} \) is the discrete trajectory costs, accounting for the cost of actively manipulating component outputs.

The transfer of electrical and thermal energy is handled in two separate energy balances connected by the efficiency loss in the turbine. Equation 1c represents the transfer of thermal energy in the form of high temperature steam between the SMR, TES, and turbine. Equation 1d takes electrical output from the turbine and combines it with the time variant inputs of wind and solar energy to meet the load. Equation 1e shows the limits on total SMR production, limited by a maximum capacity. Equation 1f considers the ramping capabilities of an SMR and Equation 1h shows the storage capacity of the TES, constrained by a maximum capacity. The capacity limit of the turbine is represented in Equation 1g. \( r_{\text{down}} \) and \( r_{\text{up}} \) are the ramping constraints (Table 2). Equation 1i sums the component manipulation costs, simulating possible expenses, and safety concerns related to component dynamics.

2.4. Model Parameters

Model parameters are uncertain economic and system parameters in Tables 1 and 2 and fixed parameters in Table 3. Exact values for the uncertain parameters are included in the model analysis as a range of possible values as they are not well-known for the proposed system. The range between the minimum and maximum values for each parameter are chosen such that the real value of the parameter should be within the range. Each of the uncertain parameters has a nominal or best-estimate value, which is the value used when the uncertainty of that parameter is not being considered and is an estimate based on literature sources as described in Section 2.5.

Economic parameters for the renewable generation resources are not the focus of this study and variation in these parameters cannot affect optimal system design or dispatch, so these parameters are not varied as part of the study. Variation in component lifetimes also is equivalent to variation in component capital costs for this model, so the component lifetimes are held fixed.

2.5. Data Sources

Due to the nature of an uncertainty analysis, perfect adherence to specific values is not required. Of greater importance for this model are results that are applicable a variety of feasible NHES scales. All parameter values in this study are chosen to resemble those found in literature. Exact values of the nominal, maximum, and minimum values are largely arbitrary, but ranges are determined based on an analysis of currently available resources.

Time series data used in this model includes load, solar, and wind profiles from the California Independent System Operator (CAISO). It is based on the period from 29 March 2020 to 1 Aug 2020. The time series data is arbitrarily scaled so that the maximum load in the first two weeks of data matches the anticipated nameplate capacity of a 6-module
Table 1: Economic Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nominal</th>
<th>Min</th>
<th>Max</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{\text{cost,smr}}$</td>
<td>2e-6</td>
<td>1e-5</td>
<td>1e-6</td>
<td>SMR manipulation penalty</td>
<td>USD/MWth/hr</td>
</tr>
<tr>
<td>$C_{\text{cap,smr}}$</td>
<td>1.1428e7</td>
<td>8.143e6</td>
<td>2.4303e7</td>
<td>SMR capital cost</td>
<td>USD/MWth</td>
</tr>
<tr>
<td>$C_{\text{foc,smr}}$</td>
<td>4.781e4</td>
<td>3.825e4</td>
<td>5.737e4</td>
<td>SMR fixed cost</td>
<td>USD/MWth-yr</td>
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<tr>
<td>$C_{\text{voc,smr}}$</td>
<td>2.39</td>
<td>0</td>
<td>5</td>
<td>SMR variable cost</td>
<td>USD/MWth-h</td>
</tr>
<tr>
<td>$D_{\text{cost,tes}}$</td>
<td>2e-6</td>
<td>1e-5</td>
<td>1e-6</td>
<td>TES manipulation penalty</td>
<td>USD/MWth/hr</td>
</tr>
<tr>
<td>$C_{\text{cap,tes}}$</td>
<td>7.171e4</td>
<td>3.134e4</td>
<td>1.3147e5</td>
<td>TES capital cost</td>
<td>USD/MWth</td>
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<tr>
<td>$C_{\text{foc,tes}}$</td>
<td>20</td>
<td>14</td>
<td>43</td>
<td>TES fixed cost</td>
<td>USD/MWth-yr</td>
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<tr>
<td>$C_{\text{voc,tes}}$</td>
<td>14</td>
<td>3</td>
<td>29</td>
<td>TES variable cost</td>
<td>USD/MWth-h</td>
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<tr>
<td>$C_{\text{cap,turb}}$</td>
<td>6.66e5</td>
<td>5.00e5</td>
<td>7.00e5</td>
<td>Turbine capital cost</td>
<td>USD/MWe</td>
</tr>
<tr>
<td>$C_{\text{foc,turb}}$</td>
<td>5.2e4</td>
<td>4.0e4</td>
<td>6.0e4</td>
<td>Turbine fixed cost</td>
<td>USD/MWe-yr</td>
</tr>
<tr>
<td>$C_{\text{voc,turb}}$</td>
<td>1.0</td>
<td>0</td>
<td>2.0</td>
<td>Turbine variable cost</td>
<td>USD/MWe-h</td>
</tr>
</tbody>
</table>

Table 2: System Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{\text{smr}}$</td>
<td>60</td>
<td>yrs</td>
<td>SMR lifetime</td>
</tr>
<tr>
<td>$L_{\text{sol}}$</td>
<td>35</td>
<td>yrs</td>
<td>Solar lifetime</td>
</tr>
<tr>
<td>$L_{\text{wind}}$</td>
<td>30</td>
<td>yrs</td>
<td>Wind lifetime</td>
</tr>
<tr>
<td>$L_{\text{Turb}}$</td>
<td>50</td>
<td>yrs</td>
<td>Turbine lifetime</td>
</tr>
<tr>
<td>$L_{\text{sys}}$</td>
<td>30</td>
<td>yrs</td>
<td>System lifetime</td>
</tr>
<tr>
<td>$C_{\text{cap,wind}}$</td>
<td>1.877e6</td>
<td>USD</td>
<td>Wind capital cost</td>
</tr>
<tr>
<td>$C_{\text{foc,wind}}$</td>
<td>3.97e4</td>
<td>USD</td>
<td>Wind fixed cost</td>
</tr>
<tr>
<td>$C_{\text{voc,wind}}$</td>
<td>1e-4</td>
<td>USD</td>
<td>Wind variable cost</td>
</tr>
<tr>
<td>$C_{\text{cap,sol}}$</td>
<td>2.534e7</td>
<td>USD</td>
<td>Solar capital cost</td>
</tr>
<tr>
<td>$C_{\text{foc,sol}}$</td>
<td>2.18e4</td>
<td>USD</td>
<td>Solar fixed cost</td>
</tr>
<tr>
<td>$C_{\text{voc,sol}}$</td>
<td>1e-4</td>
<td>USD</td>
<td>Solar variable cost</td>
</tr>
</tbody>
</table>

Table 3: Fixed Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
<th>Description</th>
</tr>
</thead>
</table>

NuScale power plant. The US Energy Information Administration provides estimates for both capital and operational costs involved with solar and wind power generation. [17].
Capital cost values for SMRs are based on 2020 capital cost estimates from NuScale [36, 35]. Operation and maintenance data from Kehlhofer et al. [28] gives reasonable values for the fixed and variable costs of an SMR. Estimates for turbine costs are taken from a Department of Energy report on combined heat and power systems [15].

Jacob et al. provided capital cost estimates for a TES system [27]. The range of TES fixed and variable operational costs are based on a report released by IRENA [26] and work done by Wagner [45]. Kuravi [31] and Alva [5] give the values used to produce the TES efficiency range.

2.6. Sources of Uncertainty

There are three sources or categories of uncertainty considered in this model: variation in design parameters, variation in length of dispatch horizon, and variation in simulated time series. The first of these is due to uncertain model parameters that remain fixed over the dispatch horizon. These parameters are expected to have a single, fixed value, but the exact value is not well-known, so a range of possible values must be considered. These parameters include the economic and design parameters as described in Tables 1 and 2.

A second source of uncertainty is in the length of time horizon used for the dispatch of the system. Due to limited computing power, it is not generally feasible to perform a detailed simulation of the dispatch over the entire system lifetime. The length of the time horizon used for the dispatch optimization can affect the optimal sizing of system components, particularly storage components.

The third source of uncertainty is the variation in the uncertain time series involved in the problem. The wind and solar generation as well as the system load at each point in time are not perfectly known ahead of time and exhibit random or imperfectly-known variation in existing power grids. The variation in each of these time series must be addressed in order to design a robust plant capable of operating under a wide range of circumstances.

The economic costs of the fixed-capacity elements of the system (wind farm, solar farm, and turbine) are not analyzed as a source of uncertainty in this model. The sizing and operation of each of these components is fixed by the problem definition. A change in these values does not affect the optimal design or operation of the system, although they would affect the resulting system LCOE.

3. Calculation

![Figure 2: Flowchart for selection of model parameters](image)
The purpose of this work is to determine which sources of uncertainty are most important when optimizing the design and operation of a simple NHES system. A number of sensitivity analysis methods are compared to determine which sources are most influential. The effects of each of the sources or categories of uncertainty are first considered independently. This means that when one category of uncertainty is being analyzed, all the other categories are held at nominal values as indicated in Figure 2. Some combined effects are also considered as discussed below. Inflation, demand-side response and market effects are not considered in this work.

3.1. Optimization Problem Formulation

The model is formulated as a combined design and dispatch optimization problem using GEKKO [8]. Combining the optimization of the design and dispatch into a single problem is a common method for efficiently sizing hybrid energy systems [12, 37].

GEKKO provides an intuitive Python algebraic modeling language with an interface to large-scale nonlinear optimizers including APOPT [22] and IPOPT [44]. GEKKO also provides algorithmic differentiation for efficiently determining model derivatives and orthogonal collocation on finite elements for discretizing differential equations. These features facilitate the clear and efficient formulation of nonlinear programming problems.

The combined design and dispatch problem is formulated with GEKKO fixed variables used to describe the design variables and GEKKO variables to define the dispatch variables. This allows the design variables to have a single optimized value over the dispatch horizon while the dispatch variables and differential equations are automatically discretized by GEKKO.

Several techniques were used to increase problem tractability and thereby facilitate more sophisticated uncertainty analysis for this model. First, the problem was formulated so as to maintain continuous first and second derivatives. This included replacing square roots and other functions with equivalent formulations that have continuous derivatives throughout the model. Second, the electrical energy balance was formulated as an inequality constraint with a penalty for overproduction rather than as an equality constraint. Both of these techniques greatly improved the model tractability and decreased the required time to solution. The resulting base model with a dispatch horizon length of 360 hours results in a nonlinear programming problem with 12930 variables and 11488 constraints that is solved with a large-scale, sparse, nonlinear programming (NLP) solver.

3.2. Monte Carlo and Sensitivity Analysis

A Monte Carlo analysis using Latin hypercube sampling (LHS) and local sensitivity analysis at each sample point provides insight into the sensitivity of the optimal system design to variations in the design parameters. A large number of LHS samples of the input parameters are generated. The model is then evaluated for each particular sample and local sensitivity is performed around each sample point using a forward difference strategy with an adaptive step size.

The sensitivity at each point is a forward difference from the sample point for each input parameter. Each of these sensitivities is normalized by both the model output at the
samples point \((Q_{j,0})\) and initial sample model input at that point \((P_{i,0})\), allowing the various sensitivities to be analyzed in terms of the percent output parameter change that is caused by a percent input parameter change. The sensitivity calculation is described mathematically in Eqn. 2 where \(S_{i,j}\) is the sensitivity of the \(i^{th}\) parameter with respect to the \(j^{th}\) output.

\[
S_{i,j} = \frac{P_{i,0}}{Q_{j,0}} \frac{dQ_j}{dP_i}
\]

The result of this analysis is a \(n \times m\) matrix of sensitivities at each sample point where \(n\) is the number of uncertain model parameters (inputs) and \(m\) is the number of model outputs.

Useful results from this analysis include the distributions of the model outputs, correlations between the models input and outputs, and distributions of the normalized sensitivities. This method analyzes the effect of a parameter variation across the design space and determines where a design space parameter is likely to have the greatest effect. Because the sensitivities are normalized, they can be compared to determine which parameters are most influential on the performance of the system and which are least influential.

3.3. Dispatch Horizon Length Variation

The length of the time series also affects the optimization results. Longer time horizons produce different optimal dispatch patterns. In particular, the usage pattern and optimal capacities of storage elements changes with the length of the dispatch horizon. The effect of NHES time horizon length is characterized by analyzing the variation of optimal LCOE, TES capacity, and SMR capacity. The purpose of this analysis is to determine what dispatch horizon length is most efficient while still maintaining optimal dispatch.

Further insight into the impact of dispatch horizon length is obtained by repeating the analysis at each distinct length with a number of LHS samples of the design parameters. This allows visualization of how the output distributions change with increasing dispatch horizon length rather than simply observing the change for the nominal case.

3.4. Time Series Forecast Variability

Variation in uncertain time series also affects the performance and optimal design of NHES. The variation in uncertain time series can be due to both predictable sources (e.g., solar radiation patterns) and currently unpredictable sources (e.g., local wind speed) resulting in time series consisting of both predictable and stochastic elements.

Systems optimally sized for a single sample of an uncertain time series may perform poorly or completely fail for other samples of the time series. This work aims to quantify the variability in system performance, optimal sizing, and feasibility due to variations in uncertain time series.

For the purposes of this work, uncertain time series are modeled using the Fast Fourier Transform (FFT) detrending and auto-regressive moving average (ARMA) time series modeling tools provided in RAVEN [3, 11]. These tools model uncertain time series as a combination of predictable and stochastic variation and allow generating large numbers of synthetic samples from the modeled uncertain time series. The generated samples can then be used to evaluate the performance of an NHES under a wide range of conditions.
In modeling the uncertain time series, the synthetic samples show a visible decrease in sample smoothness as compared to the original samples. In order to correct this, the samples are smoothed using rolling averages until the first and second auto-correlation lag coefficients as well as the standard deviation of the signal are close to the original signal [17]. An example of how the synthetic time series are generated from historical data is shown in Figure 3.

![Historical CAISO load data](image1)

![Scaling](image2)

![Stochastic Sampling using RAVEN](image3)

![Smoothing the stochastic samples](image4)

Figure 3: Example of time series generation for CAISO load data

The variability of NHES performance and optimal design due to time series uncertainty is quantified by optimizing the system for a number of different realizations of the uncertain time series. This is done for each of the uncertain time series individually (load, wind, solar) with the other time series fixed and then again with variations in all the time series. This quantifies the uncertainty due to each of the time series as well as capturing the combined effects of the variation.

4. Results and Discussion

The effect of each source of uncertainty is analyzed according to the methods described above. These effects can be ranked by the output distributions that they cause on the model outputs ($N_{SMR}$, $N_{TES}$, $N_{Turb}$, LCOE). Broader distributions indicate stronger effects while narrower distributions indicate weaker effects.
4.1. Monte Carlo and Sensitivity Analysis

Monte Carlo sensitivity analysis for 360 and 500 hour dispatch horizons quantify the model output uncertainty due to the design parameter uncertainty. A heatmap of the Pearson correlations between the LHS inputs and model outputs is shown in Figure 4. The correlation between the model inputs is not considered as they are independent uniform variables in this model. Dispatch horizons longer than 500 hours are not considered as computational demand did not allow good sampling of the input space.

The optimal turbine size is excluded as it remained essentially the same for each time horizon length and so is not correlated to any other inputs or outputs. This lack of variation in optimal turbine sizes is due to the fact that the time series are not varied in this part of the analysis, leading to a consistent maximum net system load and thus a consistent required turbine capacity. Results between the 360 and 500 hour dispatch horizons are nearly the same, so only the 500 hour dispatch results are shown.

The top five parameters with absolute Pearson correlations coefficients ($r$) greater than 0.1 are the turbine efficiency ($\eta_{turb}$), the SMR capital cost ($C_{cap,SMR}$), the TES efficiency ($\eta_{TES}$), the SMR variable cost ($C_{var,SMR}$), and the TES capital cost ($C_{cap,TES}$). The remaining design parameters had either very weak ($r \leq 0.05$) correlations to the model outputs or no correlation at all ($r = 0$). Excluding nonlinear or combined effects, this clearly emphasizes the importance of accurately determining the value of the most significant parameters over the ones with no significant correlation and greatly reduces the input space of the model.

An increase in efficiency of either sized unit is correlated with a decrease in both system LCOE and both unit capacities. The SMR and TES capital cost correlations highlight the competing relative sizes of the TES and SMR. The two components can to some extent compensate for each other and the precise sizing decision is likely an economic one. The single parameter modification most likely to improve the system LCOE would be a reduction in the SMR capital cost.

Eleven non-zero normalized sensitivities of the system are shown in Fig. 5 with the outlier values excluded. With the exception of outliers, the absolute values of the remaining normalized sensitivities are all less than 0.001. All but the two weakest of the plotted sensitivities are with respect to one of the top five influential parameters. This strengthens the conclusion that the top five parameters shown above are the most important of the
proposed parameters to correctly determine, while the exact values of the remaining eleven parameters are largely negligible.

Finally, approximations of the model output probability density functions are generated using Gaussian kernel density estimators as shown in Figure 6 with the triangles representing the distribution medians. There is a minimum value for the SMR when the design parameter uncertainty is considered. Any size below that is insufficient, but it is less likely to need one of larger capacity. All three outputs show broad distributions, indicating that variation in the modeling parameters do have a strong effect on the model output.

Both the 360 and 500 hour dispatch results are shown for comparison. The median SMR capacity increases by 7.8 $MW_{th}$ from 360 hour to 500 hour dispatch with a significant change in the shape of the distribution. With a complete change in the shape of the distribution, the median LCOE increases by 5.7 USD/MWh and the median TES capacity changes by over 10200 $MW_{th}$.

The large change in expected storage capacity indicates that expected cycle time of the
storage unit may be a key consideration in choosing the dispatch horizon length. The chosen dispatch horizon must be longer than the expected storage cycle of the longest storage element in the system.

None of the output distributions are normal distributions, which indicates the nonlinear relationship between the model inputs and outputs. Linearized approximations of the current model introduce an approximation error.

The analysis indicates that the most important parameters to accurately determine for this system are the turbine and TES efficiencies, the SMR and TES capital costs, and the SMR variable cost. None of the other parameters exhibit a large influence over the tested parameter space, but the expected variation due to design parameter uncertainty is high.

4.2. Dispatch Horizon Length Variation

The response of both the nominal case and LHS samples of the uncertain design parameters to variation in the dispatch horizon length is shown in Figure 7. The nominal case is shown as a blue line for each of the model outputs and the variation due to design parameter uncertainty is shown in the violin plots along the $x$ axis. Time series variation was not included in this part of the study.

![Figure 7: Model response to variation in dispatch time horizon](image)

There is no conclusive evidence that the dispatch horizon length is long enough to approach an infinite horizon solution. Further analysis with longer time horizons is needed, but
this is currently not feasible with this study and the limitations of compute power. These results illustrate the sensitivity of the system to the choice of dispatch horizon length.

The turbine size \( N_{\text{turb}} \) is once again fixed at each dispatch horizon length by the maximum net load in the horizon. As such, it does not have variability with the system design parameters. Including the uncertainty due to time series variation would likely cause variation in turbine size.

The high levels of variability in the nominal case are evidence that dispatch horizon length is a critical consideration in optimal design of NHES. There are ranges of dispatch horizon lengths that produce similar model responses, which indicates that there may be unique modes of operation depending on the length of time horizon being considered. For example, for cases where storage is used for daily peak-shifting, 200 hour dispatch horizons may be sufficient, while longer horizons would be required for more demanding storage schemes.

The high levels of variability in the LCOE and TES and SMR capacities at the majority of the dispatch horizon lengths indicate that, while dispatch horizon length is a critical consideration, uncertainty in the design parameters must still be considered. Uncertainty in the design parameters has a much larger effect on the LCOE and SMR capacity and a comparable effect on the TES capacity.

Most of the uncertainty in the LCOE and SMR capacity are a result of uncertainty in the design parameters, rather than the length of the dispatch horizon used in the problem while the uncertainty of the turbine and TES capacities are more affected by variations in dispatch horizon length than by design parameter uncertainty.

### 4.3. Time Series Variability

Gaussian kernel density estimator (KDE) approximations of the model output distributions for the uncertain load, wind, and solar time series are shown in Figure 8. The standard deviation and mean of each output distribution for each variable is tabulated in Table 4. The uncertain time series are examined one at a time using 1000 samples. Parameter covariance is not considered. Lack of rigorous validation in the generation of time series samples limits the conclusions that can be drawn from this study as the input samples are not guaranteed to be representative of a real system.

<table>
<thead>
<tr>
<th>Time Series</th>
<th>LCOE Std. Dev. (Mean)</th>
<th>( N_{\text{TES}} ) Std. Dev. (Mean)</th>
<th>( N_{\text{SMR}} ) Std. Dev. (Mean)</th>
<th>( N_{\text{turb}} ) Std. Dev. (Mean)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load</td>
<td>0.54 (65.52)</td>
<td>1992.08 (5597.69)</td>
<td>17.61 (1024.45)</td>
<td>12.30 (497.56)</td>
</tr>
<tr>
<td>Wind</td>
<td>0.91 (67.10)</td>
<td>1381.25 (5618.93)</td>
<td>19.22 (906.44)</td>
<td>9.88 (460.43)</td>
</tr>
<tr>
<td>Solar</td>
<td>0.38 (68.29)</td>
<td>524.61 (12697.77)</td>
<td>3.10 (893.33)</td>
<td>3.50 (438.92)</td>
</tr>
</tbody>
</table>

Table 4: Standard deviations and means of output distributions caused by time series variation

Variation in the solar generation profile produces the tightest output distribution in all four cases as well as higher mean values for both system LCOE and TES capacity. This may be due to the more predictable daylight generation of solar power and makes characterization of the solar generation profile less important for reducing output uncertainty.
than the wind and load profiles. Both wind generation and load profile uncertainty result in output distributions similar in width although different in range for each of the four cases. Wind and load profiles are of approximately equal importance for optimization results depending on the output parameter.

The combined effects of the uncertain time series, based on 10,000 samples with 495 hour dispatch horizons, are shown in Figure 9. The standard deviations of the output distributions are also shown in Table 5. All four outputs are positively-skewed, near-Gaussian distributions of varying widths and standard deviations. The individual effects of time series variation are often significantly non-Gaussian, which indicates that some variation caused by individual time series variation is not additive in this system.

Both the SMR and turbine capacity distributions are shifted higher than the distributions predicted by the individual time series variations. The distributions are not necessarily wider, but fall outside the range of the individual time series variation predictions. This leads to an important observation that the output distributions or even upper and lower bounds on the output distributions cannot be accurately predicted based on the results of individual time series variation. While the lack of validation in NHES and time series modeling limits the applicability of these results, this work highlights the importance of combined time series variation analysis.

4.4. Relative Effects

The relative effects of the various sources of uncertainty on the model outputs are shown as violin plots in Figure 10 where the type of uncertainty is listed on the X axis for each subplot.

Figure 8: Distribution of model outputs for individual time series variation
Figure 9: Distribution of model outputs for 10000 samples of simultaneous time series variation using 495 hour dispatch

Figure 10: Comparison of the output variation due to the three categories of input variation

<table>
<thead>
<tr>
<th>Variation Source</th>
<th>LCOE Std. Dev. (Mean)</th>
<th>$N_{TES}$ Std. Dev. (Mean)</th>
<th>$N_{SMR}$ Std. Dev. (Mean)</th>
<th>$N_{turb}$ Std. Dev. (Mean)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>22.68 (91.01)</td>
<td>3586.49 (11522.71)</td>
<td>79.00 (893.00)</td>
<td>0.00 (437.17)</td>
</tr>
<tr>
<td>Dispatch length</td>
<td>1.24 (66.16)</td>
<td>5776.22 (13570.32)</td>
<td>0.00 (873.30)</td>
<td>59.30 (491.38)</td>
</tr>
<tr>
<td>Time Series</td>
<td>0.97 (69.41)</td>
<td>1875.56 (5185.27)</td>
<td>26.02 (1104.66)</td>
<td>14.73 (526.86)</td>
</tr>
</tbody>
</table>

Table 5: Standard deviation (rounded) of each of the output distributions for each source of uncertainty

The effect of parameter variation on system LCOE is at least an order of magnitude greater than either of the other sources of uncertainty. Parameter characterization is therefore the most important method for reducing uncertainty in the system LCOE. The three sources of uncertainty produce similar ranges of values for the TES capacity with dispatch
horizon length and time series variation being respectively the most and least influential sources. Parameter variation has the largest effect on the SMR capacity with time series variation having a significant effect and dispatch horizon length having little to no effect. Parameter uncertainty has no real effect on the turbine capacity while dispatch horizon length has the largest effect and time series variation has a significant effect.

As noted in Section 4.1, effects of parameter variation on turbine capacity are not fully represented as the turbine capacity is fixed when both the time series and dispatch horizon length are fixed. This is due to the design of the sample NHES system. Other systems may exhibit different relative effects of the sources of uncertainty.

4.5. Summary

Overall, the impact of the various considered uncertainties varies widely, particularly when this impact is broken down by model output.

Dispatch horizon length is important, but does not have a large effect on the optimal SMR capacity or the system LCOE. For this particular case study, it does have a large effect on the TES and turbine capacities. Depending on the design details of the NHES under consideration, some sources of uncertainty may be negligible.

Variations in the wind power production and load signals are independently more influential than variations in solar generation. When considered together, the combined effects from the time series variation are not always predictable based on the individual effects. Proper analysis requires a combination of multiple uncertain time series.

The relative impact of each of the sources of uncertainty depends on the model output. The SMR capacity and LCOE are most affected by design parameter uncertainty, while the TES is affected by all three sources, and the turbine capacity is most affected by the dispatch length. This understanding of the relative importance of each of the parameters can help drive model accuracy improvements in an efficient manner.

Based on the standard deviation of the output distributions, the parameter uncertainty has the largest impact in 2 of the 4 outputs and dispatch horizon length was the largest in the remaining 2. Time series variation was the least influential source of uncertainty in the proposed system.

5. Conclusions

Understanding the effects of parameter, time series, and modeling uncertainties is critical to furthering the development of NHES. A proper understanding of these uncertainties increases the accuracy of modeling results and can speed model development and analysis by excluding insignificant uncertainties. While the numerical results of this work are not broadly applicable to all NHES, the methodology presented in this paper is useful for a broad range of systems. There are a number of useful conclusions for this system that characterize the insights gained by this methodology and may be applicable to other NHES.

First, of the 16 design parameters considered only 5 have significant impact on the system performance which allows for model reduction. Similar model reductions may be possible for a wide range of NHES models.
Second, different design variables of an NHES can be impacted very differently by the same uncertainty. Some uncertainties, or sources of uncertainty, may be negligible for one design variable while being highly influential for another design variable. This analysis demonstrates methods of comparing dissimilar uncertainties, and sources of uncertainties, to determine which are most important for a given model.

Finally, this analysis demonstrates that uncertain time series are more accurately handled together rather than individually. Isolating the effects of individual uncertain time series can ignore important interactions between dissimilar time series.

Areas of future work include analyzing the impact of linear approximations when modeling NHES systems and analyzing the impact of time series forecasting on NHES performance.

6. Data Availability

The input data, Python scripts for analysis, and plotting as well as the analysis results are available in the following Git repository: https://github.com/BYU-PRISM/NHES-opt-sensitivity

Additional information about the available data and scripts are available in the Readme.md file at the root of the repository.

7. Future Work

The accuracy of this work is limited by the short dispatch horizons used to allow for computationally reasonable models. More work is needed with longer dispatch horizons, which may require linearized models for the infinite horizon arrival cost. In this case, this approximation needs to be examined as well.

Additionally, this work is confined to a single NHES operating independently to meet a required electrical demand. Further work is required to analyze how such a system responds when operating as part of a larger electrical power grid in both regulated and deregulated energy markets. Additional work is also needed to determine how generalizable the results of this study are to NHES generally.

Finally, limited parameter and operational data is available due to the proprietary nature of commercial operations and the lack of existing NHES. Repeating or modifying this work with more accurate model parameters and operation data would further clarify the role that each of the involved uncertainties has on system performance.

8. Acknowledgements

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References


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