Constrained Control and Optimization of Tubular Solid Oxide Fuel Cells for Extending Cell Lifetime

Benjamin Spivey  
ExxonMobil

John Hedengren  
Brigham Young University

Thomas Edgar  
The University of Texas at Austin

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Objective:

Perform load-following and disturbance rejection with tubular solid oxide fuel cells while operating within thermal stress-based constraints. Multi-input multi-output (MIMO) linear model predictive control combines controlled variables for power, thermal stress, and other failure modes into one performance index.

Agenda:

• Description and Motivation of SOFC Power Generation
• Dynamic Modeling of Thermal Stress Indicators
• Constrained Predictive MIMO Control of the SOFC
• Conclusion
Description and Motivation of SOFC Power Generation
SOFC Operational Principles

Electricity is primarily produced through $\text{H}_2$ oxidation. CO oxidation also occurs.

Tubular SOFC Systems

Methane is internally reformed given operating temperatures from 600-1000°C.
Why the interest in SOFCs?

• High efficiencies over a wide power range (1 kW – 100 MW):
  40-50% (LHV) for SOFC, 60-70% for GT-SOFC, 80-90% for GT-SOFC + cogeneration.

• Fuel flexibility:
  – Natural gas, gasoline, diesel, coal
  – Hydrogen, methanol, ethanol, biomass

• Suitability for cogeneration with high exhaust temperatures
• Low noise and emission levels.

Why are SOFCs not in widespread use?

• Reported lifetimes have yet to reach goals – 40,000 h (DOE) – causing cost of electricity to be high.
• Microcracking, sulfur catalyst poisoning, carbon deposition, and air & fuel starvation decrease lifetime.
Dynamic Modeling of Thermal Stress Indicators
### SOFC Performance and Operational Variables

<table>
<thead>
<tr>
<th>Performance Requirement</th>
<th>Controlled Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC Power Delivery</td>
<td>Power (W)</td>
</tr>
<tr>
<td>Thermal Stress Minimization</td>
<td>Minimum Cell Temperature (K)</td>
</tr>
<tr>
<td></td>
<td>Radial Thermal Gradient (K/m)</td>
</tr>
<tr>
<td>Avoid Carbon Deposition</td>
<td>Steam-to-Carbon Ratio</td>
</tr>
<tr>
<td>Avoid Air and Fuel Starvation</td>
<td>Air and Fuel Utilization (%)</td>
</tr>
</tbody>
</table>

Fischer (2009) reports that minimum stack temperature and radial thermal gradient are the primary two contributors to high tensile thermal stresses. Conclusion agrees with Nakajo (2006).
SOFC System Model in Simulink

Next Slides: SOFC model details
The University of Texas at Austin

Benjamin James Spivey

SOFC Submodel: 2D Model Discretization

SOFC Cross-Section in Radial (r) and Axial (x) Directions

Total DAE States per Radial Element  = 65: Temperatures, Molar Flows, Current, and Intermediate Variables

Total Nodes per Steady-State Model  = 40, per Dynamic Model  = 10

The distributed parameter model captures factors causing high thermal stresses
SOFC Submodel: First-Principles Equations

Electrochemical Model

\[ V_{\text{cell}} = V_{\text{oc}} - \eta_{\text{act}} - \eta_{\text{conc}} - \eta_{\text{ohm}} \]
\[ V_{\text{oc}} = V_{H_2}^0 + \frac{RT}{2F} \ln \left( \frac{P_{H_2}}{P_{H_2}^0} \right) \]
\[ V_{H_2}^0 = -\frac{\Delta G_0}{2F} + \frac{\Delta S_0}{2F} (T - 298) \]

\[ \eta_{\text{ohm}} = I R_{\text{ohm}} \quad \eta_{\text{act}} = \frac{RT}{nF} \frac{i}{i_0} \]
\[ i_{0,\text{an}} = \gamma_{\text{an}} \left( \frac{P_{H_2}}{P_{\text{amb}}} \right) \left( \frac{P_{H_2}^0}{P_{\text{amb}}} \right)^m \exp \left( -\frac{E_{\text{act,an}}}{RT} \right) \]
\[ i_{0,\text{cat}} = \gamma_{\text{cat}} \left( \frac{P_{O_2}}{P_{\text{amb}}} \right)^{0.25} \exp \left( -\frac{E_{\text{act,an}}}{RT} \right) \frac{A}{m^2} \]

Energy Conservation Model

\[ \rho V c_{p,i} \frac{dT_{s,i}}{dt} = \]
\[ h A (T_{s,\text{sur}} - T_{s,i}) + k A \frac{dT_{s,i}}{dx} + \varepsilon F \sigma A (T_{s,\text{opp}}^4 - T_{s,i}^4) \]
\[ Q_{\text{elec}} = \left( \frac{\Delta H_f,H_2O(g)}{n F} - V_{\text{cell}} \right) i, \]
\[ \frac{c_{p,ig}}{R} = \alpha + \beta T + \gamma T^2 + \frac{\zeta}{T^2} \]

Steam Methane Reforming Model

\[ CH_4 + H_2O \rightleftharpoons \frac{r_{CH_4}}{} CO + 3H_2 \]
\[ CO + H_2O \rightleftharpoons \frac{r_{\text{shift}}}{p_{H_2}} CO_2 + H_2 \]
\[ r_{CH_4} = A \exp \left( -\frac{E_a}{RT} \right) p_{CH_4} \]
\[ r_{\text{shift}} = k \left( X_{H_2O} X_{CO} - \frac{X_{H_2} X_{CO_2}}{K_{eq}} \right) \]
\[ K_{eq} = \exp \left( -0.2935 \zeta^3 + 0.635 \zeta^2 + 4.1788 \zeta + 0.3169 \right) \]
\[ \zeta = \frac{1000}{T} - 1. \]

Some past literature iterate b/t electrochemical and energy models for steady-state solutions – here it is solved simultaneously and dynamically using APMonitor Modeling Language.
SOFC Submodel: Radiation Sensitivity

Radiation Analysis for Plant B: Air channel radiation is significant

**Without Radiation**

**With Radiation**

**Radiation Effects:**
- Increased peak temperature
- Inlet air and solid PEN is closer in temperature
- Molar flow exhibits negligible change.

**Final Steady-State Model** = Validated Campanari Model + Air Channel Radiation
SOFC System Model in Simulink

Next Slides: Dynamic results for full SOFC system model
Dynamic Response of Controlled Variables

Power

- Power increases as fuel flow rate/density increases – with higher pressure and lower temperature.
- Voltage or current is not an effective MV for load-following near peak power.

Maximum Radial Thermal Gradient

- Main MVs for controlling power also affect the thermal gradient significantly.
- Air temperature has little effect due to insulation from the thick cathode.
- The gradient responds on the same time scale as power due to numerator dynamics.

New Conclusion - thermal gradients should be controlled at the same time scale as the power
Dynamic Response of Controlled Variables

Axial Cell Temperatures

Fuel pressure and temperature effect on inlet cell temperature is twice or greater than middle or outlet temperature.

Air temperature effects outlet temp but not inlet. Relatively thick cathode (1 mm) insulates the anode inlet.

Key Conclusions:
1) The median temperature dynamics are unique from the minimum temperature dynamics.
2) Some variables may be disturbances to the minimum temperature if not set as MVs – inlet fuel temperature.
3) The primary MVs for load-following also affect the minimum temperature.
Constrained Predictive MIMO Control of the SOFC
MIMO Control Structure

Decentralized MIMO + SISO Control

Tracked CVs
- Power
- $T_{cell,min}$

Constrained CVs
- $d_{Trad,max}$
- $T_{cell,min}$
- SCR
- $U_f$
- $U_a$

Measurements or estimates of the thermal stress indicators are used directly as controlled variables.
Linear MPC Algorithm

MPC Formulation:
Constrained Nonlinear Programming Optimization

\[
\min_{u(t)} J = \frac{1}{2} (x - x_{ref})^T \tilde{Q} (x - x_{ref}) + \frac{1}{2} \Delta u^T R \Delta u + \frac{1}{2} \xi^T V \xi
\]

where

\[
x - x_{ref} = (\Gamma \Delta u + \Omega x_0) - \tilde{C} y_{ref}
\]

s.t.

\[
\begin{align*}
& u_{\min} \leq u(k) \leq u_{\max} \\
& \Delta u_{\min} \leq \Delta u(k) \leq \Delta u_{\max} \\
& y_{\min} \leq y(k) + \xi(k) \leq y_{\max}
\end{align*}
\]

Unique Features
- Tracking and slack variable weights vary by variable and time step to control variables with very different dynamics and settling times.
- Minimum cell temperature has a time constant 100-1000x greater than the fast time constant of other CVs.

Vectors represent variable values across the horizon
Reference Trajectory

Reference trajectory converges to the CV target with first-order dynamics.

$$y_{ref} = y_0 + y_{sp} \left( 1 - \exp \left( -\frac{t}{\tau_r} \right) \right)$$

The time constant, $\tau_r$, is defined as $1/5$ of the acceptable time to reach steady-state.

Analytical Derivatives

Analytical derivatives are used by the MATLAB `fmincon` solver to reduce computational time. Derivatives are calculated using matrix calculus.

$$\frac{dJ}{du} = \begin{bmatrix} \frac{dJ}{du}, \frac{dJ}{d\xi} \end{bmatrix}$$

$$\frac{dJ}{du} = \left( (x - x_{ref})^T \tilde{Q} \Gamma + \Delta u^T R \right) \frac{d\Delta u}{du}$$

$$\frac{dJ}{d\xi} = \xi^T V$$

Kalman Filter State Estimation

The Kalman filter provides the linear unbiased minimum variance estimate for the unmeasured states.

$$X^+(k)_j = X^-(k)_j + \Delta x(k)_j$$

$$\Delta x(k)_j = K(k)_j \Delta y(k)_j$$

$$\Delta y(k)_j = Y(k)_j - G(k)_j$$

$$G(k)_j = C_j X^-(k)_j$$

Kalman gain

$$K_j = \tilde{P}(k)_j \tilde{H}_j \left( \tilde{H}_j \tilde{P}(k)_j \tilde{H}_j^T \right)^{-T} + R$$

$$\tilde{H}_j = C_j,$$
Objective: Determine whether controlling outlet gas temperatures or average cell temperature provides control of thermal stress indicators – common approach in literature.

Minimum Temperature Control

- Power output settles in 400 s.
- The minimum cell temperature settles to the target in 700 s.
- Minimum cell temperature constraint prevents limit violation.
- The minimum cell temperature and steam-to carbon ratio limits affect the solution at t=150 s.

Load-following is achieved while maintaining thermal stress indicators in limits.
MPC Results: Reliability Control Study

Average Temperature Control

- Power and average temperature are tracked. No other CVs are constrained.

- The minimum temperature travels 30 K past the previous lower limit constraint. Outlet temperature control results are nearly identical.

- Steam-to-carbon ratio would violate limits without constrained control.

Nearly identical results when outlet gas temperature is controlled.
MPC Results: Load-Following Study

Objective: Test load-following while satisfying thermal-stress based constraints.

- Load-following is achieved between 250-280 kW with settling times between 50 – 750 s.
- Minimum cell temperature, thermal stress constraints, and stream-to-carbon ratio lengthen settling time at higher loads.
MPC Results: Disturbance Rejection Study

Objective: Investigate whether LMPC maintains power output in the presence of alternating fuel quality, relevant for biogas applications.

- An augmented MV variable is used to include disturbance measurements.

\[
\Delta \tilde{u}(k) = \begin{bmatrix} \Delta u(k) \\ \Delta d(k) \end{bmatrix}
\]

- Load demand is satisfied within 2% of the setpoint.

- Three CVs are riding constraints with negligible violations at low methane conditions.

Disturbance Variable: CH$_4$ mole %
MPC Results: Disturbance Rejection Study

Manipulated Variable Plots

3 of 4 manipulated variables are at constraint values as methane decreases from peak values.
Conclusions

Dynamic Modeling of Thermal Stress Indicators

- Investigated dynamics of minimum cell temperature and maximum radial thermal gradient for the tubular SOFC.
- Thermal stress should be controlled on same time scale as load due to numerator dynamics of thermal gradient.
- Demonstrated effect of radiation on tubular SOFC temperature and concentration profiles.

Constrained Predictive MIMO Control of the SOFC

- A distributed-parameter based model is recommended for controlling thermal stress indicator dynamics. Lumped models or outlet temperature measurements are not sufficient.
- Varying time constants of SOFC can be accommodated by using non-constant MPC tracking weights.
- LMPC is capable of rejecting measured fuel quality disturbances.
Acknowledgments

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• Prof. John Hedengren
• Dr. Dunia
• APMonitor Modeling Language
• ExxonMobil
• NSF IGERT Program
• Labmates
Questions ?
Appendix
Problem Statement

Research is needed to investigate the dynamics of factors causing the SOFC to experience shortened lifetimes, especially microcracking, and directly control these factors.
# SOFC System Modeling Decisions

<table>
<thead>
<tr>
<th>Feature</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 Molar Gas Species</td>
<td>7 species (H2, H2O, N2, O2, CH4, CO, CO2) are needed to accommodate methane fuel and air oxidant. Some models may use 3 (H2, H2O, O2).</td>
</tr>
<tr>
<td>2D Discretization</td>
<td>Axial and radial discretization is required to capture <em>minimum cell temperature</em> and <em>maximum radial thermal gradient</em>. 0D (lumped) and 1D models capture neither and have less accurate performance prediction.</td>
</tr>
<tr>
<td>Voltage Losses</td>
<td>Includes ohmic, activation, and diffusion losses. Some models include only 1.</td>
</tr>
<tr>
<td>Material Properties</td>
<td>Temperature-dependent, nonlinear ohmic resistance and specific heat models.</td>
</tr>
<tr>
<td>Pressure Drop</td>
<td>Based on Darcy’s law, compressible flow with &lt; 10% pressure drop. Models may choose constant pressure drop.</td>
</tr>
<tr>
<td>Minimum/Maximum Functions</td>
<td>Variables may occur at different locations – maximum gradient, minimum temperature.</td>
</tr>
<tr>
<td>Multiple Submodels</td>
<td>SOFC, Ejector, Prerreformer. Necessary for modeling real inputs.</td>
</tr>
<tr>
<td>Time Delays</td>
<td>Transport time delays since molar transport is assumed at quasi-steady-state</td>
</tr>
</tbody>
</table>

**Goal:** accurate dynamic model directly applicable to real SOFC system operation.
SOFC and Balance of Plant

Several MVs and DVs enter through ejector

Ejector and prereformer models are necessary to accommodate realistic MVs

Ejector: Quasi-Steady-State Algebraic Model
9 States
Matlab

Prereformer: Quasi-Steady-State Algebraic Model
8 States
Matlab

SOFC: Dynamic DAE Model
220 States
APMonitor

MV: manipulated variable
CV: controlled variable
DV: disturbance variable

Cell Temperatures

Air/Fuel Utilization
Steam-Carbon Ratio

Power & Efficiency

Variable Speed Compressor

Fuel

Air

Fuel Tanks

PC

TC

MV

FC

TT

TT

TT

TT

PC

MV

PC

MV

MV

CV

FC

MV

CV

TC

MV

SOFC Exhaust

Ejector and prereformer models are necessary to accommodate realistic MVs
SOFC System Simulink Model

SOFC S-Function

Ejector S-Function

Time Delay

Prereformer S-Function

Disturbance Module

 Benjamin James Spivey

The University of Texas at Austin
Simulation Time Discretization: Power Response to Voltage Step

- Decreasing time steps below 1 s yields little change in dynamic response.
- The QSS gas transport assumption is valid to 1s time steps.

Transport Time Delays

- Delays are important for sub-60 s response.
- Transport delay is a function of mass flow.
# Model Validation and Verification

<table>
<thead>
<tr>
<th>Validation Process</th>
<th>Verification Process</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Steady-State Model</strong></td>
<td><strong>Dynamic Model</strong></td>
</tr>
<tr>
<td>1) <strong>Ensure credibility of model equations published in literature.</strong> Model is sourced from many authors due to incomplete or inaccurate models in literature.</td>
<td>1) <strong>Add energy balance dynamics to account for thermal time constant.</strong></td>
</tr>
<tr>
<td>2) <strong>Literature search for design parameters.</strong> 2D model requires many specific parameters from many authors.</td>
<td>2) <strong>Compare open-loop settling time, dynamic characteristics, and MV-CV gains to other SOFC models.</strong> Results seen in both single-step test and staircase test.</td>
</tr>
<tr>
<td>3) <strong>Match model output directly to empirical and simulation data.</strong> Only used 3 tuning parameters – heat transfer coefficient, cell outer diameter, and contact resistance. Authors may not describe theirs.</td>
<td><strong>Verification is challenging because public validation data is scarce.</strong> Noted by other authors (Bhattacharrya, 2010).</td>
</tr>
</tbody>
</table>

Steady-state model validation is consistent with the leading SOFC models in literature (Campanari, 2004; Stiller, 2006).
SOFC Submodel: Steady-State Validation

Validation: Match to Simulation Data

Spivey Model

Campanari Model

The mean absolute percentage (MAP) error is used to compare the models.

MAP for Electrolyte Temperature = 3.85%

$$MAP = \frac{1}{n} \sum_{t} \left| \frac{A_{t} - P_{t}}{A_{t}} \right|$$
SOFC Submodel: Steady-State Validation

Validation: Match to Simulation Data

Comparison of the concentration profiles also indicates that the steady-state model matches well versus the standard models used for tubular, high-temperature SOFC modeling.
Radial versus Axial Temperature Gradient

The radial gradient is negative near the fuel inlet placing the anode in tension. The radial gradient is several times the axial gradient.

Simulation results agree with prior studies indicating that radial thermal gradients are most significant.
SOFC System Model: Dynamic Model Verification

Open-loop response to fuel pressure step

Numerator dynamics is expected due to multiple time constants.

OL settling time of 2500-12000 s is consistent with literature (Hall, 1999).

Higher fuel flow increases power but decreases minimum temp – a higher reaction rate of steam-methane reformation
Dynamic Response of Controlled Variables

Fuel Utilization

Voltage has linear effect on fuel utilization – voltage changes current directly, thereby changing fuel consumption.

Changing fuel flow rate (pressure or temperature) does not affect fuel utilization greatly because it also affects power.

Steam-to-Carbon Ratio (SCR)

Adding fuel (increasing pressure or decreasing temperature) causes the steam-to-carbon ratio to decrease.

Increasing system pressure increases SCR – more recycle flow.

Decreasing voltage increases SCR – higher rate for electrochemical reaction with H₂O product.
Linear MIMO Control Model Identification

Test Signals for Dynamic Excitation
- Generated a restricted and variable length random walk signal.
- MVs and CVs are normalized by bias and slope.

Transfer Function Model

\[ Y(s) = G(s) U(s) \]
\[ G(s) = \left[ \frac{K(\tau_0 s+1)}{(\tau_1 s+1)(\tau_2 s+1)} \cdots \frac{K(\tau_n s+1)}{(\tau_1 s+1)(\tau_2 s+1)} \right] \]

Staircase tests used to constrain model parameter estimation – sign of gain, presence of numerator dynamics.

State-Space Model

\[ x_{n_x}(k+1) = A_{n_x} x_{n_x}(k) + B_{n_x} u(k) \]
\[ y(k) = C_{n_x} x_{n_x}(k). \]

Conversion from transfer function to state-space is necessary for efficient MPC matrix calculations

Process knowledge + numerical parameter estimation \(\rightarrow\) model identification
Eliminating Steady-State Offset

An unmeasured fuel quality step disturbance is applied at $t = 5s$.

**Augmented State-Space Model with Integration**

The augmented model with integration produces a negligible steady-state offset.

\[ y(k) = \begin{bmatrix} 0_{nx} & I_{ny} \end{bmatrix} \begin{bmatrix} \Delta x_{nx}(k) \\ y(k) \end{bmatrix} \]

\[ \begin{bmatrix} \Delta x_{nx}(k+1) \\ y(k+1) \end{bmatrix} = \begin{bmatrix} A_{nx} & 0_{nx,ny} \\ C_{nx}A_{nx} & I_{ny} \end{bmatrix} \begin{bmatrix} \Delta x_{nx}(k) \\ y(k) \end{bmatrix} + \begin{bmatrix} B_{nx} \\ C_{nx}B_{nx} \end{bmatrix} \Delta u(k) \]

Measurement feedback is key to eliminate offset.
MPC Results: Reliability Control Study

Reliability Study Implications:

- A control model based on distributed parameters is necessary since controlling an average SOFC temperature does not sufficiently constrain minimum temperature dynamics.

- Control based on a thermocouple measuring the outlet gas temperature would not be sufficient for containing thermal stress indicators.

- Constrained control is necessary for ensuring that the CVs do not exceed operability limits.
Disturbance Rejection Study Implications:

- Load-following is achieved despite 3 CVs and 3 MVs riding constraints at different times due to ± 10% fuel quality variation.

- The controller is capable of reacting to prevent temperature excursions even though the thermal settling time is 2500s +.

- Constrained MVs suggest improvements for system design given these disturbances – sizing an air compressor to handle lower pressures.
SOFC Optimization Subject to Operability Constraints
Steady-State Constrained Design Optimization

Objective

Find a minimal cost design and set of operating conditions for a given power demand profile of a load-following system.

Economic Optimization Model

\[
C_{\text{tot}} = C_{\text{cap}} + C_{\text{op}},
\]
\[
C_{\text{op}} = c_f V_f N_h,
\]
\[
C_{\text{cap}} = C_{\text{fin}} + C_{\text{mai}} + C_{\text{ins}},
\]
\[
C_{\text{pur}} = C_{\text{sofc}} + C_{\text{inv}} + C_{\text{pre}} + C_{\text{aux}}.
\]
\[
C_{\text{sofc}} = n_{\text{cells}} \pi D_{\text{outer}} L (2.96 T_{\text{sofc}} - 1907)
\]
\[
C_{\text{inv}} = 10^5 \left( n_{\text{cells}} \frac{P_{\text{max}}}{500} \right)^{0.7}
\]
\[
C_{\text{pre}} = 130 \left( \frac{A}{0.093} \right)^{0.78} + 3240 V_{pr}^{0.4} + 21280.5 V_{pr},
\]

Optimality defined as minimum annual system cost

\[
\min_{u(t)} J = C_{\text{tot}}
\]

\[
s.t. \quad g(x,u) = 0
\]
\[
\quad u_{\text{min}} \leq u(k) \leq u_{\text{max}}
\]
\[
\quad y_{\text{min}} \leq y(k) \leq y_{\text{max}}
\]

The first-principles SOFC system model with 2000+ states and intermediate variables permits inclusion of real CV limits.
Optimization Algorithm for Load-Following

Identify load profile PDF(s)

Choose power outputs

Solve for optimal SOFC designs and Total Cost curves

Calculate the Annual Cost Expectation for a chosen design and load profile

Optimality Criterion

Decision Variables:
SOFC size (1)
Control MVs (4)

Power demand is probabilistic for load-following applications and requires calculation of expectation values for optimization.
- Optimization results in cost savings of $5300-6100 or 5% of operating costs.

- Majority of cost savings is achieved through finding optimal operating conditions for load following.

Design optimization is achieved simultaneously with constraint satisfaction.
Summary of Contributions
Contributions

SOFC Optimization Subject to Operability Constraints

- Using a detailed, first-principles system model for optimization ensures operational feasibility of optimum.
- Demonstrated a probabilistic approach for load-following system optimization.
- Design optimization reduces operating costs by five-percent at the steady-state value while satisfying control constraints.
Publications and Conferences

  Published September 2010

• **Spivey, Edgar.** Dynamic Modeling, Simulation, and MIMO Predictive Control of a Tubular Solid Oxide Fuel Cell. *Journal of Process Control*.
  Submitted for Review in June 2011

• **Spivey, Edgar.** First-Principles-Based NMPC for a Tubular Solid Oxide Fuel Cell.
  To be submitted


SOFC Submodel: Modeling Challenges

- Distributed parameter approach produces a large number of states: 650 states and intermediates for 10 finite volumes.
- Dynamic system of differential and algebraic equations to be solved simultaneously (without algebraic loops).
- Algebraic equations are in an implicit form.
- Nonlinearities introduced by reaction and electrochemical terms.
- Multiple time scales varying from milliseconds to hours.

The complete SOFC model is solved simultaneously via constrained NLP using the APMonitor Modeling Language.
The SS SOFC model results (model) are compared to the actual plant data (expected). The model error is also compared to the Campanari model error.

<table>
<thead>
<tr>
<th>Plant A</th>
<th>Model</th>
<th>Expected</th>
<th>Error</th>
<th>Campanari Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single cell power output (W)</td>
<td>109.0</td>
<td>104.8</td>
<td>4.0</td>
<td>1.6</td>
</tr>
<tr>
<td>Current density (A/m²)</td>
<td>1792.0</td>
<td>1800.0</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Voltage (V)</td>
<td>0.7294</td>
<td>0.6981</td>
<td>4.5</td>
<td>1.6</td>
</tr>
<tr>
<td>Fuel utilization (%)</td>
<td>68.4</td>
<td>69.0</td>
<td>0.8</td>
<td>0.6</td>
</tr>
<tr>
<td>Air utilization (%)</td>
<td>17.3</td>
<td>17.8</td>
<td>2.6</td>
<td>1.7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Plant B</th>
<th>Model</th>
<th>Expected</th>
<th>Error</th>
<th>Campanari Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single cell power output (W)</td>
<td>158.0</td>
<td>157.0</td>
<td>0.6</td>
<td>3.0</td>
</tr>
<tr>
<td>Current density (A/m²)</td>
<td>3000.0</td>
<td>3000.0</td>
<td>0.0</td>
<td>1.1</td>
</tr>
<tr>
<td>Voltage (V)</td>
<td>0.6315</td>
<td>0.6275</td>
<td>0.6</td>
<td>3.0</td>
</tr>
<tr>
<td>Fuel utilization (%)</td>
<td>70.1</td>
<td>69.0</td>
<td>1.6</td>
<td>2.9</td>
</tr>
<tr>
<td>Air utilization (%)</td>
<td>23.8</td>
<td>23.8</td>
<td>0.1</td>
<td>1.7</td>
</tr>
</tbody>
</table>

The model validation error is comparable to the Campanari model error.
SOFC, ejector, and prereformer models are solved at each time step to investigate MV-CV relationships
Current and Overpotential Plot
Hall – Tubular SOFC Thermal Time Constant
SOFC Electrical Characterization

SS Electrical Characterization with Fixed Fuel Flow Rate (Plant B)

Plant B: 267 kW, 3.50 bar

- LHV efficiencies are 45% and 38% for Plants A and B respectively - typical for 100-300 kW SOFC.
- Nominal efficiency is based upon provided inputs, not plant modeling. 

\[
\eta = \frac{I \cdot V}{LHV_{H_2O} \cdot N_{H_2O,in} + LHV_{CO} \cdot N_{CO,in} + LHV_{CH_4} \cdot N_{CH_4,in}}
\]
Example Staircase MV Profile
Discretization Comparison

![Graph showing temperature comparison with different discretizations.](image)

- **Legend:**
  - 40 volumes
  - 10 volumes

- **Axes:**
  - **Y-axis:** Temperature (°C)
  - **X-axis:** Position (cm)
Fuel Quality Disturbance

+/- 15% variation in power
MPC Results: Reliability Control Study

Outlet Temperature Control

![Graphs showing outlet temperature control results.](image)