

# Optimal Trajectory Generation using Model

## Predictive Control for Aerially Towed Cable Systems

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This paper studies trajectory generation for a mothership that tows a drogue using a flexible cable. The contributions of this paper include model validation for the towed cable system described by a lumped mass extensible cable using flight data, and optimal trajectory generation for the towed cable system with tension constraints using model predictive control. The optimization problem is formulated using a combination of the squared-error and  $L_1$ -norm objective functions. Different desired circular trajectories of the towed body are used to calculate optimal trajectories for the towing vehicle subject to performance limits and wind disturbances. Trajectory generation for transitions from straight and level flight into an orbit is also presented. The computational efficiency is demonstrated, which is essential for potential real-time applications. This paper gives a framework for specifying an arbitrary flight path for the towed body by optimizing the action of the towing vehicle subject to constraints.

### Nomenclature

$A$	cross-sectional area of the cable, $\text{m}^2$
$\mathbf{a}_m$	mothership acceleration in the inertial frame, $\text{m}^2/\text{s}$
$\mathbf{c}_y, \mathbf{c}_u$	cost weights of $\mathbf{y}$ and $\mathbf{u}$
$\mathbf{d}$	parameters or unmeasured disturbances

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$E$	Young's modulus, GPa
$\mathbf{e}_{hi}, \mathbf{e}_{lo}$	slack variables
$\mathbf{f}$	equations of motion
$\mathbf{F}_{dr}^{aero}$	aerodynamic forces acting on the drogue, N
$\mathbf{F}_j^{aero}$	aerodynamic forces acting on the $j^{\text{th}}$ cable link, N
$\gamma_a$	air mass referenced flight path angle, rad
$\mathbf{G}_{dr}$	gravity of the drogue, N
$\mathbf{g}_{iec}$	inequality constraints
$\mathbf{G}_j$	gravity of the $j^{\text{th}}$ cable link, N
$h_0$	desired constant altitude of the drogue, m
$\ell_0$	unstretched length of each cable link, m
$m_{dr}$	mass of the drogue, kg
$m_j$	mass of the $j^{\text{th}}$ cable link, kg
$N$	number of cable links
$\mathbf{p}_{dr}(t)$	actual drogue position at time $t$ in the inertial frame, m
$\mathbf{p}_{dr}^d(t)$	desired drogue position at time $t$ in the inertial frame, m
$\Phi$	objective function value
$\phi$	bank angle, rad
$\mathbf{p}_j$	position of the $j^{\text{th}}$ cable joint in the inertial frame, m
$\mathbf{p}_m$	mothership position in the inertial frame, m
$\psi$	heading angle, rad
$r_{dr}^d$	desired orbit radius of the drogue, m
$\mathbf{sp}_{hi}, \mathbf{sp}_{lo}$	higher and lower dead-band setpoints
$t_0$	starting time of the simulation, s

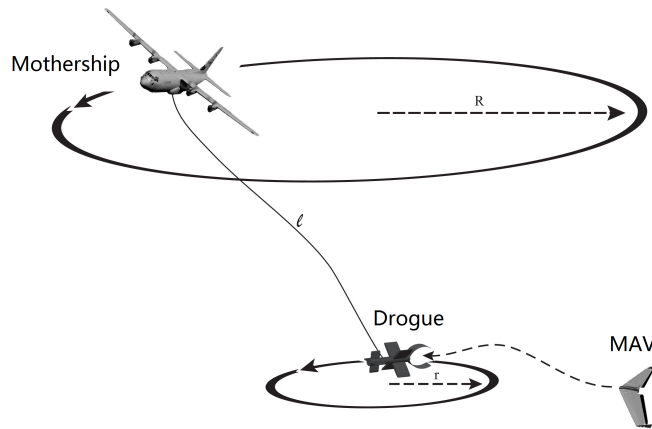
$t_1$	ending time of the simulation, s
$t_a$	starting time of the transition, s
$\tau$	response time constant of the desired controlled variables
$\theta$	orbital angle, rad
$\mathbf{T}_j$	cable tension exerted on the $j^{\text{th}}$ joint by the $(j - 1)^{\text{th}}$ joint, N
$T_p^a$	drogue orbital period with constant airspeed, s
$T_p^g$	drogue orbital period with constant ground speed, s
$\mathbf{u}$	inputs
$V_a$	magnitude of mothership airspeed, m/s
$v_{dr}^a$	magnitude of the desired drogue airspeed, m/s
$v_{dr}^g$	magnitude of the desired ground speed of the drogue, m/s
$\mathbf{v}_m$	mothership velocity relative to the wind frame, m/s
$\mathbf{w}_{hi}, \mathbf{w}_{lo}$	weights on the controlled variables outside the dead-bands
$\mathbf{x}$	states of the equations of motion
$\mathbf{y}$	controlled variables
$\mathbf{y}_{hi}, \mathbf{y}_{lo}$	higher and lower dead-bands of the controlled variables

## I. Introduction

Miniature Air Vehicles (MAVs), which are characterized by relatively low cost, superior portability, and in some cases, improved stealth, have the potential to open new application areas and broaden the availability of Unmanned Aircraft System (UAS) technology. MAVs are typically battery powered, hand launched and belly landed, and therefore may not require a runway for take-off or landing. Backpackable MAVs can be used in gathering time-critical and over-the-hill intelligence, surveillance and reconnaissance (ISR) information. However retrieving the MAV may be problematic because landing the vehicle near the operator could disclose his/her location. Another potential application of MAVs is collecting disaster damage information. Again for this application, retrieval

of the MAV after it has performed its mission is difficult because target locations are often inaccessible, and the MAV may not have enough fuel to return to its home position. The relatively low cost of MAVs suggests that they may be expendable, thereby removing the need for recovery. However, even if the costs are low, MAVs still contain critical and often classified technology which needs to be kept out of enemy hands. One option is to destroy the MAV or damage the electronics so that it cannot be reused or reverse engineered. However, most of the solutions that have been proposed require additional payload on the MAV. Cost considerations and the potential that MAV technology could fall into enemy hands will limit the use of this technology.

This paper supports the development of a potential aerial recovery system as shown in Figure 1, where the towing vehicle (mothership) enters an orbit designed to cause the towed body (drogue) to execute an orbit of smaller radius and lower speed (less than the nominal speed of the MAV). The MAV then enters the drogue orbit at its nominal airspeed and overtakes the drogue with a relatively low closing speed. There are many challenges to solve before this concept becomes feasible. In this paper we focus on optimal, open-loop trajectory design for the towing vehicle to place the drogue into a specified orbit. Additional work on various aspects of this problem can be found in [1–8].



**Fig. 1 A potential solution to the aerial recovery problem.**

The system shown in Fig. 1 is a typical circularly towed cable-body system which has been studied since D. Bernoulli (1700-1782) and L. Euler (1707-1783), who focused on the study of linearized solutions of a whirling string. Modern studies began with Kolodner [9] who made a detailed mathematical study of the free whirling of a heavy chain with a fixed tow-point. In subsequent

decades, the study of towed cable systems focused on the analysis of equilibrium and stability of the system, and on dynamic modeling approaches for the cable when the motion of the towing point is a straight line or circular orbit [10–13]. In the recent years, Williams et al. [14–17] have made major contributions to the study of towed body systems. Williams and Trivailo [14, 15] give a detailed description of the dynamics of circularly towed drogues and design strategies for moving from one orbit configuration to another. Williams and Ockels [16] employ this approach to the problem of lifting payloads using multiple fixed-wing aircraft. Williams [17] also presents a numerical approach to mitigate the disturbance of a crosswind on the periodic solution of the cable tip using a combination of towing vehicle manipulation and cable length regulation.

For a towed cable system, a mathematical representation that compromises between complexity and accuracy is essential for further studies like developing control strategies. The central problem in describing the motion of the towed cable system is the modeling of the cable. In this paper, as recommended in [18], a finite element approach is used to model the cable, which is treated as a series of  $N < \infty$  rigid links with lumped masses at the joints. Researchers have developed the equations of motion for towed cable systems using Lagrange’s method [19, 20] and Kane’s equations [21, 22], which do not scale well for a large number of links. Newton’s second law is a fundamental and widely used tool to formulate equations of motion for dynamical systems. However, this method is seldom used to establish the equations of motion for the cable in the literature. In this paper, Newton’s second law is employed to derive the equations of motion for a flexible and elastic cable.

In previous studies of towed cable systems, experimental results were used to validate the mathematical model in the simulation [4, 22–26]. Cochran et al. [23] experimentally validated the theoretical model in a wind tunnel by comparing the lateral motions of the towed body in both experimental and simulation results. Short cables (1.5 – 3 m) and different wind speed conditions were used. Borst et al. [24] compared the drogue altitude and tension forces in flight test and simulation results in which the towing plane flew in a circular orbit and a five mile long cable was used. Hover [25] conducted the experiment in a test tank using a 1000 m long cable to study the control strategy of dynamic positioning of a towed pipe under water. Clifton et al. [26] conducted a flight test by commanding the towing plane on a circular path using a 20,000 ft long cable connected

to the drogue. The drogue altitude variations were compared between flight test and simulation results. Williams et al. [22] presented experimental results using a rotating arm in a water tank towing different types of cable. Additional measurements were also taken using a 3 m long cable attached to a ceiling fan spinning at 72 rpm.

The experiments presented in the literature were conducted either using short cables, less than 10 m [22, 23], or long cables, more than 1000 m [24–26], and the previous aerial towed cable systems, typically being towed by manned aircraft, made the experiments very expensive and difficult to execute and repeat. In previous related work [4] on aerial recovery, an unmanned towing vehicle and 100 m long cable were used to collect data for model validation. The purpose was to determine aerodynamic lift and drag coefficients for the drogue in the simulation by using the model of a single-link cable. In the current paper, the model validation in [4] is extended by comparing trajectories of the drogue in flight test with those in simulations using models with different numbers of cable links. Increasing the number of cable links in the model leads to a more realistic representation of cable dynamics but also increases the computational burden. One of the objectives of this paper is to determine the number of cable links that leads to a sufficiently accurate model while allowing efficient optimal trajectory generation of the mothership.

Given a mathematical model with sufficient fidelity, a strategy is needed to regulate the mothership motion so that the drogue trajectory follows a desired path. Existing methods for generating the desired trajectory for the mothership can be classified into two categories: differential flatness based methods [1, 3, 6, 19, 27] and optimal control based methods [14, 17, 21, 28–30].

Murray [19] presented a differential flatness based solution in which the motion of the system was parametrized using the motion of the towed-body as the flat output. However, Murray’s solution technique had numerical stability problems and was not further developed. A similar scheme for using differential flatness for motion planning of the mothership was discussed by Williams [27]. In the previous work of Sun et al. [3, 6], differential flatness was applied to generate the desired trajectory for the mothership and a nonlinear control law was developed for the mothership based on its dynamic model in the presence of wind disturbances. The differential flatness based method is typically applied to the discretized model of the cable, and is computationally inexpensive com-

pared to optimal control methods. However it requires the equations of motion of the system to be differentially flat [31]. Another limitation is that this method does not take the performance limitations of the system into consideration, so that the resulting trajectory of the towing vehicle might be impractical. In particular, the resulting trajectories may violate constraint limits on the manipulated variables (e.g. maximum available mothership thrust) or the controlled variables (e.g. tension limitations of the cable).

Optimal control based methods were also used to generate the desired trajectory for the towing vehicle. Williams [14, 21] employed an optimal control method to find a periodic path for the towing vehicle in order to minimize the motion of the towed body subject to dynamic constraints. Sequential quadratic programming was used to solve the optimization problem. Williams et al. [15] used simulated annealing to solve the optimal control problem in scheduling the orbit radius of the towing vehicle while the system transitions from a straight flight into an orbit. Williams et al. [32] used optimal control in determining the motion of the towing plane, as well as the cable deployment rate so that the towed body passed through a set of desired waypoints. Williams [17] extended his work to find an optimal elliptical orbit and cable deployment rate to compensate for crosswind disturbances. Establishing an optimal motion of the towing vehicle subject to constraints using a discretized multi-link cable model is a complicated optimization problem with many states and degrees of freedom. However, discussions of the computational burden were seldom mentioned. The typical algorithm used in solving the problem are based on quadratic programming in which squared-error objectives are used. In this paper, an approach based on model predictive control (MPC) using the  $L_1$ -norm and squared error objectives are introduced and applied to perform the optimal trajectory generation of the towing vehicle.

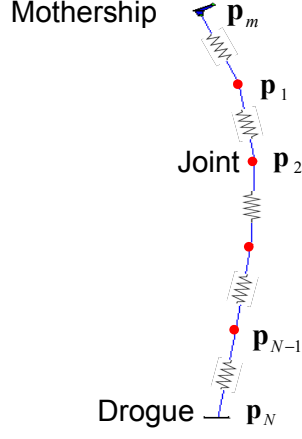
MPC has been widely used in industrial applications such as chemical plants and refineries [33] based on empirical linear models [33–35] obtained by system identification. Because many of these applications have either semi-batch characteristics or nonlinear behavior, the linear models are retrofitted with elements that approximate nonlinear control characteristics to ensure that the linear models are applicable over a wider range of operating conditions and disturbances. The linear models adapt with either time or as a function of the current state of the system. A more general

form that is not dependent on model switching is collections of differential and algebraic equations (DAEs) in open equation format [36]. These equations may include equality or inequality constraints, integer variables, and differential elements [37]. To apply DAEs to nonlinear models, different approaches have been studied and implemented in the literature, including simultaneous methods [38], decomposition methods [39, 40], efficient nonlinear programming solvers [41], improved estimation techniques [42–45], and large-scale techniques for applications to industrial systems [46, 47]. The objective function used in the control optimization problems are typically based on a weighted squared error or an  $L_2$ -norm form.

One novel contribution of this paper is the employment of a new  $L_1$ -norm objective function in the optimal control problem. The  $L_1$ -norm objective has a number of advantages over traditional squared-error or  $L_2$ -norm objectives, including less sensitivity to data outliers and better rejection of measurement noise. Many of the remaining challenges associated with implementing nonlinear models are due to the complexity of the numerical solution techniques. To meet this demand, commercial and academic software has been developed. APMonitor Modeling Language [48] is one of the software packages that aim to model and solve the large-scale DAEs. Many algorithms like filtered bias updating, Kalman filtering, moving horizon estimation (MHE) and nonlinear MPC can be implemented in this web-services platform through interfaces to MATLAB or Python. In this paper, a nonlinear MPC method is used to solve the optimal trajectory generation problem in which the mothership with performance limits is maneuvered to place the towed drogue onto a desired orbit. A combined objective functions is utilized in which a squared-error objective is used to quantify the trajectory tracking error and  $L_1$ -norm objectives are employed to regulate the constraints. The MPC algorithm is implemented using APMonitor Modeling Language.

The remainder of the paper is structured as follows. In Section II, the mathematical model of the cable-drogue system is established using a lumped mass approach and Newton’s second law is employed to derive the dynamic equations of the system. In Section III, the mathematical model of the cable using different numbers of links is validated using flight test data. Section IV introduces the formulation of nonlinear MPC used to generate a constrained optimal trajectory for the mothership. Section V shows simulation results using the resulting optimal trajectories. The analysis of the flight





**Fig. 2** The mothership-cable-drogue system in the simulation.

test results is presented in Section VI.

## II. Mathematical Model of the Cable-drogue System

The cable connecting the mothership and drogue can be modeled as an elastic or non-elastic flexible string. In the literature, the dynamics of towed-body systems were modeled by assuming that the cable is flexible and non-elastic [2, 19, 23, 26, 49, 50]. In previous flight tests, a fishing line was utilized as the cable in which considerable stretch was observed [4]. An elastic model for the cable is therefore needed in simulation to properly capture the dynamics of the system. In this section, the cable-drogue dynamics is derived using an elastic model. Figure 2 depicts a mothership-cable-drogue system with an  $N$ -link cable modeled as a finite number of point mass nodes ( $\mathbf{p}_1$  to  $\mathbf{p}_N$ ) connected by  $N$ -link springs. The drogue ( $\mathbf{p}_N$ ) is considered as the last joint of the cable. The mothership ( $\mathbf{p}_m$ ) is also modeled as a point mass.

From Newton's second law, the equations of motion of the  $j^{\text{th}}$  cable joint and the drogue are given by

$$m_j \ddot{\mathbf{p}}_j = \mathbf{T}_j + \mathbf{G}_j + \mathbf{F}_j^{aero} - \mathbf{T}_{j+1}, \quad j = 1, 2, \dots, N-1,$$

$$(m_N + m_{dr}) \ddot{\mathbf{p}}_N = \mathbf{T}_N + \mathbf{G}_N + \mathbf{F}_N^{aero} + \mathbf{G}_{dr} + \mathbf{F}_{dr}^{aero},$$

where  $\mathbf{p}_j \in \mathbb{R}^3$  is the position of the  $j^{\text{th}}$  joint in the inertial frame,  $\mathbf{p}_m \in \mathbb{R}^3$  is the position of the mothership,  $m_j$  and  $m_{dr}$  are the mass of the  $j^{\text{th}}$  link and the drogue respectively,  $\mathbf{T}_j$  is the tension,

$\mathbf{G}_j$  is the force of gravity, and  $\mathbf{F}_j^{aero}$  is the aerodynamic forces acting on the  $j^{\text{th}}$  link, and where  $\mathbf{G}_{dr}$  is the gravity force and  $\mathbf{F}_{dr}^{aero}$  is the aerodynamic forces acting on the drogue. The tension exerted on the  $j^{\text{th}}$  mass by the  $(j-1)^{\text{th}}$  mass is given by

$$\mathbf{T}_j = \frac{EA}{2\ell_0} (\|\mathbf{p}_{j-1} - \mathbf{p}_j\| - \ell_0 + \|\mathbf{p}_{j-1} - \mathbf{p}_j\| - \ell_0) \frac{\mathbf{p}_{j-1} - \mathbf{p}_j}{\|\mathbf{p}_{j-1} - \mathbf{p}_j\|},$$

where  $E$  is the Young's modulus,  $A$  is the cross-sectional area of the cable, and  $\ell_0$  is the unstretched length of each link. Detailed expressions for the gravity and aerodynamic forces are given in [14].

### III. Validation of the Mathematical Model using Experimental Data

In this section, the fidelity of the mathematical model is validated with flight data. Because the number of cable links used in the simulation determines the complexity of the equations of motion of the cable and affects the computation time in the optimization algorithms described in Section IV, this section focuses on determining an appropriate number of cable links that strikes a compromise between the accuracy and complexity of the model.

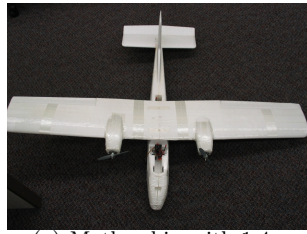
#### A. Hardware System Description

The hardware system used to collect experimental data consisted of four elements: a mothership UAS, a hemisphere-shaped drogue, a 100 m long cable and a ground station. The key parameters of the system are shown in Table 1.

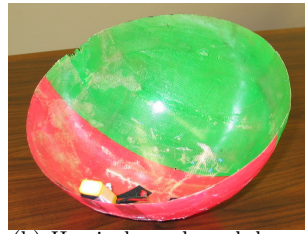
**Table 1 System parameters in flight test**

Mothership		Drogue		Cable	
Mass (kg)	1.76	Mass (kg)	0.32	Mass (kg)	0.02
$C_L$	0.28	Wing area (m <sup>2</sup> )	0.055	Length (m)	85
$C_D$	0.06	$C_L$	0.01	$E$ (GPa)	1.9
Wing area (m <sup>2</sup> )	0.307	$C_D$	0.42	$d$ (mm)	0.46
Wing span (m)	1.4	Diameter (m)	0.3		

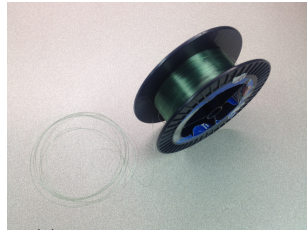
The mothership, shown in Fig. 3 (a), is a fixed wing UAS with two 770 Watt battery-operated motors, and is equipped with a Kestrel 2 autopilot, shown in Fig. 3 (d), and a radio modem to communicate with the ground station. To prevent the cable and drogue from exerting large forces



(a) Mothership with 1.4 m wing span, twin props and cable release mechanism.



(b) Hemisphere-shaped drogue with 30 cm diameter made of reinforced plastic.



(c) Fishing line cable with 20 lb maximal payload.

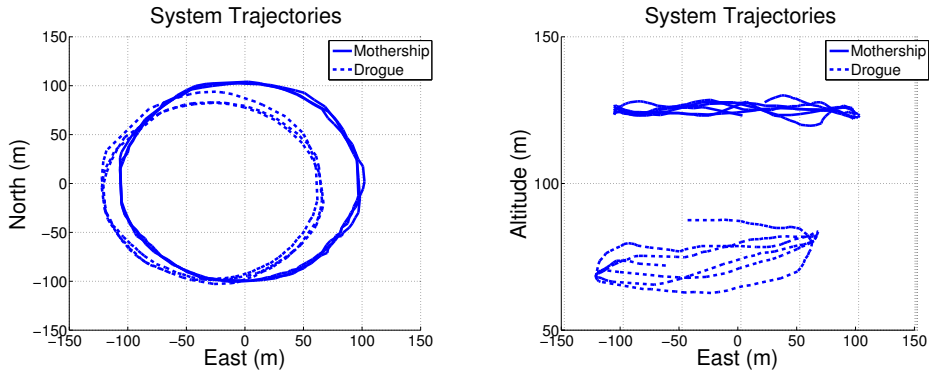


(d) Kestrel 2 autopilot developed by Procerus Technologies.

**Fig. 3 Hardware systems used in flight test.**

during the landing phase, a cable release module was placed on the underside of the mothership, and was actuated from the ground station. The hemisphere-shaped drogue with 30-cm diameter used in the flight test was constructed of reinforced plastic, as shown in Fig. 3 (b). The drogue was equipped with a Kestrel 2 autopilot and radio modem for reporting its position and velocity to the ground station. The cable is a trichloroethylene fishing line, with 0.46 mm diameter and 20 lb maximal load. The mass of a 100 m cable is approximate 20 g. The ground station consists of a desktop computer with Intel *i5* processor running at 3.1 GHz with 8 GB RAM, a radio modem communication box, and a remote controller. The ground station control software was Virtual Cockpit (VC), developed by Procerus Technologies.

We should point out that for an actual aerial recovery scenario, the airspeed of the mothership will likely be significantly greater than the airspeed of the MAV, which will require a much longer (more than 200 m) cable. However, our purpose in this section is not to demonstrate aerial recovery, but is to validate the dynamic equations of motion for the mothership-cable-drogue system. Due to limited time and budget, all flight tests were performed using battery powered RC aircraft with limited airspeed and limited payload capacity. Therefore, a relatively short cable length was used.



(a) Top-down view of the orbits of the mothership and drogue in flight test.

(b) Side (East-altitude) view of the orbits of the mothership and drogue during flight test.

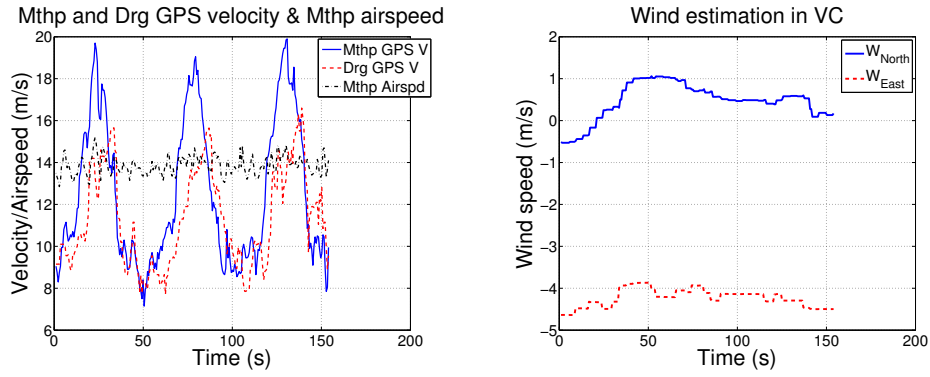
**Fig. 4 Trajectories of the mothership and drogue in the flight test.**

## B. Flight Test

In the flight experiments, the mothership was commanded to follow a loiter of 100 m radius and a constant altitude of 125 m with the airspeed commanded at 14 m/s. The results of the system trajectory are shown in Fig. 4. The top-down view of the system trajectory presented in Fig. 4 (a) shows that a circular mothership orbit resulted in a smaller circular orbit of the drogue. Because of the wind, the center of the drogue orbit shifted to the west. The East-Altitude view of the system trajectory presented in Fig. 4 (b) shows that the resulting drogue orbit was inclined because of the wind. The amplitude of the drogue’s altitude oscillation was approximately 20 m. The onboard measurement of GPS velocities of the mothership and drogue and the airspeed of the mothership are shown in Fig. 5 (a). It can be seen that the actual airspeed of the mothership essentially followed the commanded value, and the GPS velocities of the mothership and drogue oscillated between 8 m/s and 20 m/s, which implies the average wind was approximately 6 m/s. Figure 5 (b) shows the wind estimation in the north and east directions, respectively. The direction of the wind matched the direction of the center shift of the drogue orbit, while the average magnitude of the wind (approximately 4 m/s) was smaller than that implied in Fig. 5 (a)

## C. Model Validation

To validate the mathematical model, the simulated mothership is forced to follow the same trajectory as the actual mothership, and then the motion of the simulated drogue is compared to



(a) Mothership airspeed (dash-dot line), the GPS velocities of the mothership (solid line) and drogue (dashed line), respectively. (b) Wind estimation in North (solid line) and East (dashed line) directions.

**Fig. 5 Measurements obtained from Virtual Cockpit.**

the motion of the actual drogue. The number of cable links in the simulation is increased until a suitable match is obtained.

In the simulation, the airspeed of the mothership was selected as 14 m/s and the constant wind vector was selected as  $(0.5 - 4)$  m/s in the North-East coordinate. Figure 6 overlays the mothership trajectories from both flight test and simulation in North, East and altitude directions, respectively. It can be seen that the trajectory of the simulated mothership essentially matched the trajectory of the actual mothership during the flight test.

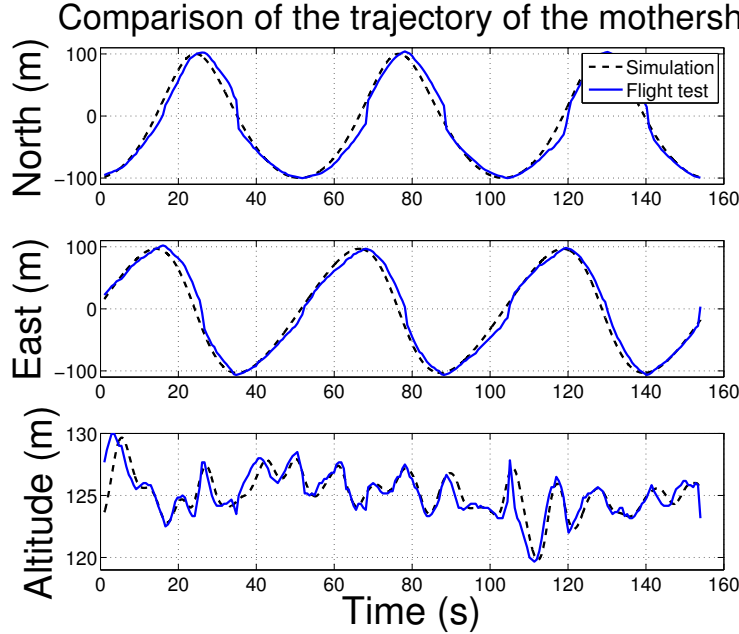
Figure 7 and 8 show the top-down and side views of the drogue trajectories from both simulation and the flight test using 1, 2, 20 and 30 cable links, respectively. It can be seen from Fig. 7 that as the number of cable links increases, the radius of the simulated drogue orbits increases to match the real drogue orbit more precisely in the horizontal direction, while it can be seen from Fig. 8 that the amplitudes of simulated drogue's altitude oscillations decreased which deviate from the flight test result in the vertical direction. Therefore, it can be seen that only increasing the number of cable links is not sufficient to make the simulated results match the flight test results in both horizontal and vertical directions.

The plots labeled as "Drg<sub>N=20</sub><sup>\*</sup>" in both Figs. 7 and 8 show the result where the number of cable links was selected as 20, and the aerodynamic drag coefficient of the drogue was selected as 0.6 instead of 0.42 with all the other parameters the same as in Table 1. It can be seen that

although the simulated result of “ $\text{Drg}_{N=20}^*$ ” matches the flight test result more precisely in the vertical direction than the one using 0.42 as the aerodynamic drag coefficient of the drogue, the radius of the orbit in the top-down view (close to the orbit using 2-link cable) deviates from the real drogue trajectory in the horizontal direction with respect to “ $\text{Drg}_{N=20}$ ”.

Thus, given a set of experimental data, it is nontrivial to determine key parameters (e.g., the ones in Table 1) as well as the number of cable links so that the simulated results match the flight test results exactly. There are numerous real life phenomena that are not captured in the model of the mothership-cable-drogue system, including wind gusts, and non-homogenous atmosphere. The discrepancy in the longitudinal direction is most likely explained by these unmeasured environmental factors.

It can be seen from Figs. 7 and 8 that the results using 2-link model match the actual data sufficient well. Because this paper focuses on the design of optimal mothership trajectories, and as a compromise between the accuracy of the simulation model and the computational burden of the optimal control calculation, the number of cable links was selected as  $N = 2$  for the trajectory design.



**Fig. 6** The mothership trajectories of the simulation (dashed line) and the flight test (solid line) in three dimensions, respectively.

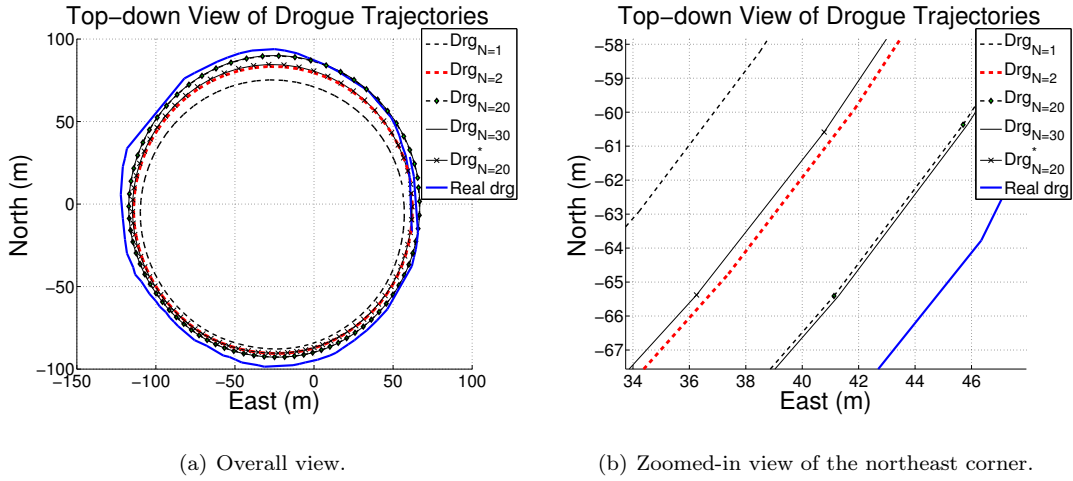


Fig. 7 Top-down view of drogue trajectories in the flight test and simulation using a different number of cable links.

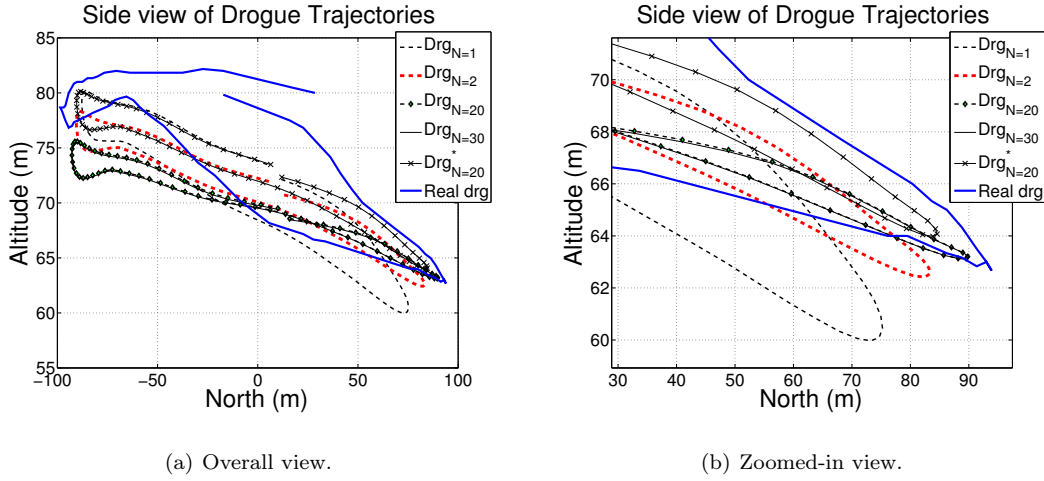


Fig. 8 Side view of drogue trajectories in the flight test and simulation using a different number of cable links.

#### IV. MPC Formulation

Given a desired trajectory for the drogue (e.g., a level circular orbit), a strategy is needed to generate a mothership trajectory that produces the desired drogue path. Differential flatness has been used in the trajectory generation of the towed cable system in [3, 6, 19, 27] by applying certain types of dynamic models of the cable. For the mathematical model presented in Section II, it is nontrivial to calculate the desired mothership trajectory by using differential flatness. In addition, since the mothership has performance limits like airspeed, roll angle, and climbing rate, differential flatness based methods do not directly take these constraints into consideration. Furthermore,

during transitions between straight and level flight and orbital flight, the tension forces exerted on the cable should not exceed the loading limit of the cable.

In this section, a strategy is developed to produce an optimal trajectory for the mothership to place the drogue onto the desired orbit in the presence of system constraints. In the literature, when formulating the optimal control problem for generating the desired mothership trajectory, the nonlinear kinematic or dynamic models of the mothership are typically used, and the objective functions are usually based on squared error or  $L_2$ -norm form. In this section, to reduce the computation time, linear dynamic equations of the mothership and a novel format of the objective function are used to formulate the MPC problem.

The mothership acceleration in the inertial frame is selected as the input to the mothership dynamic, and the equations of motion for the mothership are given by

$$\dot{\mathbf{p}}_m = \mathbf{v}_m + \mathbf{w}_c \quad (1)$$

$$\dot{\mathbf{v}}_m = \mathbf{a}_m, \quad (2)$$

where  $\mathbf{w}_c$  is the wind velocity. The performance constraints of the mothership are typically given by the magnitude of the mothership airspeed  $V_a$ , the heading angle  $\psi$ , the bank angle  $\phi$ , and the air mass referenced flight path angle  $\gamma_a$ , which is defined as the angle from the inertial North-East plane to the velocity vector of the aircraft relative to the air mass. The kinematic equations of motion for the mothership are written as

$$\dot{p}_n = V_a \cos \psi \cos \gamma_a + w_n \quad (3)$$

$$\dot{p}_e = V_a \sin \psi \cos \gamma_a + w_e \quad (4)$$

$$\dot{p}_d = -V_a \sin \gamma_a + w_d \quad (5)$$

$$\dot{\psi} = \frac{g}{V_a} \tan \phi. \quad (6)$$



By comparing Eqs. (1) and (2), the constrained variables can be expressed as

$$V_a = \|\mathbf{v}_m\| \quad (7)$$

$$\psi = \tan^{-1} \left( \frac{\mathbf{v}_m(2)}{\mathbf{v}_m(1)} \right) \quad (8)$$

$$\gamma_a = -\sin^{-1} \left( \frac{\mathbf{v}_m(3)}{V_a} \right) \quad (9)$$

$$\dot{\psi} = \frac{\mathbf{a}_m(2) \mathbf{v}_m(1) - \mathbf{a}_m(1) \mathbf{v}_m(2)}{\|\mathbf{v}_m\|^2} \quad (10)$$

$$\phi = \tan^{-1} \left( \frac{V_a \dot{\psi}}{g} \right). \quad (11)$$

The slack variables [51]  $\mathbf{e}_{hi}$  and  $\mathbf{e}_{lo}$  are selected by the optimizer to penalize  $\mathbf{y}$  above and below the dead-band, and are given by

$$\mathbf{e}_{hi,i} = \begin{cases} \mathbf{y}_i - \mathbf{y}_{hi,i} & \mathbf{y}_i - \mathbf{y}_{hi,i} \geq 0 \\ 0 & \mathbf{y}_i - \mathbf{y}_{hi,i} < 0 \end{cases} \quad \text{and} \quad \mathbf{e}_{lo,i} = \begin{cases} \mathbf{y}_{lo,i} - \mathbf{y}_i & \mathbf{y}_{lo,i} - \mathbf{y}_i \geq 0 \\ 0 & \mathbf{y}_{lo,i} - \mathbf{y}_i < 0 \end{cases}, \quad i = 1, \dots, n.$$

The trajectory generation problem can be posed as the following optimization problem:

$$\begin{aligned} \min_{\mathbf{u}(t_0, t_1)} \Phi &= \mathbf{w}_{hi}^T \mathbf{e}_{hi} + \mathbf{w}_{lo}^T \mathbf{e}_{lo} + \mathbf{y}^T \mathbf{c}_y + \mathbf{u}^T \mathbf{c}_u \\ &+ \int_{t_0}^{t_1} (\mathbf{p}_{dr}(\delta) - \mathbf{p}_{dr}^d(\delta))^T (\mathbf{p}_{dr}(\delta) - \mathbf{p}_{dr}^d(\delta)) d\delta \end{aligned} \quad (12a)$$

$$\text{s.t. } \mathbf{f}(\dot{\mathbf{x}}, \mathbf{x}, \mathbf{y}, \mathbf{u}, \mathbf{d}) = \mathbf{0} \quad (12b)$$

$$\mathbf{g}_{iec}(\dot{\mathbf{x}}, \mathbf{x}, \mathbf{y}, \mathbf{u}, \mathbf{d}) \geq \mathbf{0} \quad (12c)$$

$$\tau \frac{\partial \mathbf{y}_{hi}}{\partial t} + \mathbf{y}_{hi} = \mathbf{s} \mathbf{p}_{hi} \quad (12d)$$

$$\tau \frac{\partial \mathbf{y}_{lo}}{\partial t} + \mathbf{y}_{lo} = \mathbf{s} \mathbf{p}_{lo}. \quad (12e)$$

A combination of  $L_1$ -norm and squared-error objectives, shown in Eq. (12a), is used to accomplish multiple objectives. The controlled variables  $y$  are selected as the constraints of the system. The  $L_1$ -norm objective was used to regulate high-priority constraints like the cable tension and the airspeed of the mothership. In this case, the controlled variables were not forced to follow a desired trajectory, but were constrained to remain within a certain range of acceptable limits. The slack variables  $\mathbf{e}_{hi}$  and  $\mathbf{e}_{lo}$  are then used to regulate  $\mathbf{y}$  to remain within dynamic dead-bands parametrized by  $\mathbf{y}_{hi}$  and  $\mathbf{y}_{lo}$ . The squared-error (integration) term was used to penalize the trajectory tracking

error of the drogue with a lower weighting that represented the lower priority of the tracking objective. Eqs. (12b) and (12c) are used to regulate the states to satisfy the equations of motion and inequality constraints.

Eqs. (12d) and (12e) are linear first order equations that define the regulations for controlled variables represented by either a dead-band or reference trajectory to the setpoints. The setpoints  $\mathbf{sp}_{hi}$  and  $\mathbf{sp}_{lo}$  are used to define regions that are not penalized in the objective function and are referred to as the "dead-band". It is desirable to make the evolution of the controlled variables effectively approach setpoints at a specified rate so that excessive movements of manipulated variables or response overshoots of controlled variables can be avoided. Based on different control objectives, the initial conditions of  $\mathbf{y}_{hi}$  and  $\mathbf{y}_{lo}$  can be set to give a wider dead-band at the beginning of the simulation, and to only enforce the steady state response (and vice versa). Different initial conditions defining a wide or narrow dead-band are the trade-off between precise steady-state response and precise dynamic evolution. Therefore, reference trajectories of  $\mathbf{y}$  can be selected by specifying the values of  $\mathbf{sp}_{hi}$  and  $\mathbf{sp}_{lo}$  as a step, a ramp, or another dynamic signal.

## V. Numerical Results

In this section, the MPC based approach described in Section IV is employed to compute open-loop mothership trajectories for a variety of desired drogue trajectories. The performance limits of the mothership are selected as  $\dot{\psi} \in [-0.35, 0.35]$  rad/s,  $V_a \in [10, 20]$  m/s and  $\gamma_a \in [-0.35, 0.35]$  rad. The MPC problem was solved using APMonitor Modeling Language [48]. A step size of 2 seconds was selected as a compromise between the computation time and the accuracy of the results. The computer used to solve the optimization problem has an AMD 64 core processor with 64 GB of RAM. The trajectory design was conducted off-line and the resulting trajectories were stored for further retrieval.

### A. Desired Drogue Orbit with Constant Ground Speed

In the final phase of the aerial recovery scenario, the drogue must be placed onto an orbit that can be easily followed by the MAV. In this section, the desired drogue trajectory is a circular orbit with constant altitude and constant ground speed. Let  $r_{dr}^d$  be the desired constant orbit radius

of the drogue,  $\theta(t)$  be the orbital angle of the orbit measured from North, and  $h_0$  be the desired constant altitude of the drogue, the desired circular orbit of the drogue in three dimensions is given by

$$p_{dr_n}^d(t) = r_{dr}^d \cos \theta(t) \quad (13)$$

$$p_{dr_e}^d(t) = r_{dr}^d \sin \theta(t) \quad (14)$$

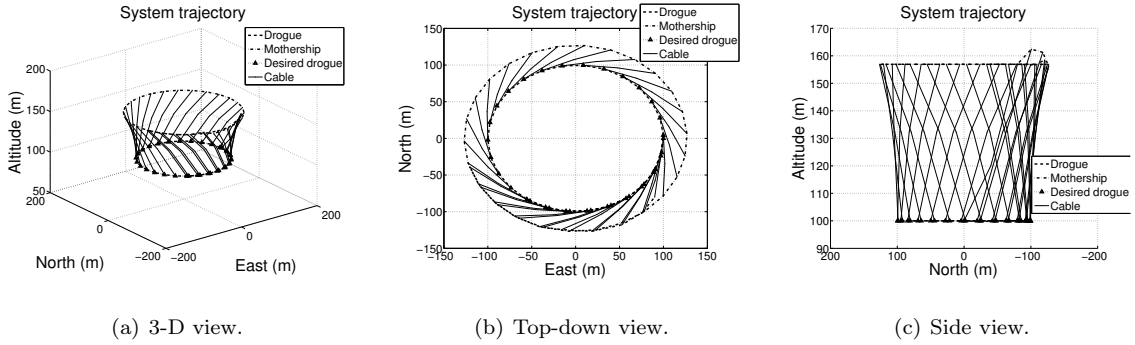
$$p_{dr_a}^d(t) = -h_0. \quad (15)$$

The orbit angle for a clockwise motion can be written as  $\theta(t) = v_{dr}^g t / r_{dr}$ . Without loss of generality, the wind is assumed to be from the west. A typical circular orbit for the drogue can be parametrized by selecting  $v_{dr}^g = 12$  m/s,  $h_0 = 100$  m and  $r_{dr}^d = 100$  m. Then the orbit period is  $T_p^g = 2\pi r_{dr}^d / v_{dr}^g = 52.36$  s. The starting and ending times are selected as  $t_0 = 0$  and  $t_1 = 70$  s, so that the resulting mothership trajectory has enough waypoints to produce an orbit. The optimal control solver is selected as IPOPT [52], which is an open-source Interior Point solver for solving Nonlinear Programming (NLP) problems included with the COIN-OR collection. The initial configuration and solution results are shown in Table 2. It can be seen that the objective function values were less than 0.5 when the wind speeds were less than 5 m/s, and increased to approximately 8000 when the wind speed increased to 10 m/s. This is because the existing performance limits of the mothership made the resulting optimal orbit unable to precisely place the drogue into the desired orbit.

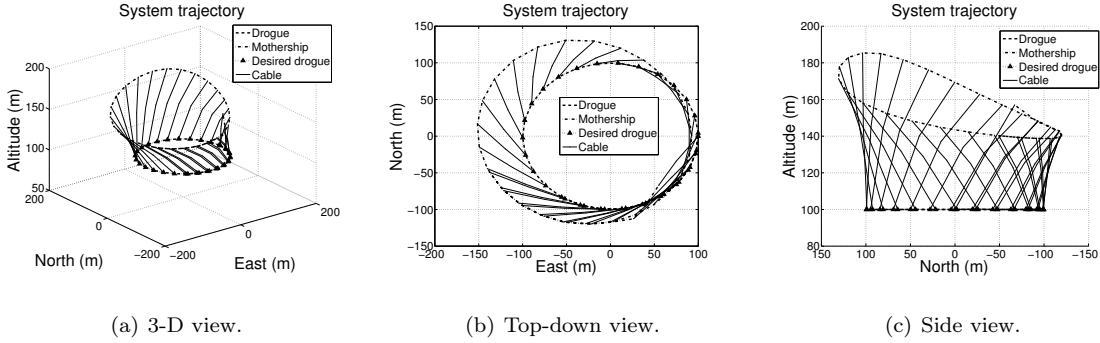
**Table 2 Solution results using the desired drogue orbit with constant ground speed.**

Wind speed (m/s)	$\mathbf{p}_m^0$ (m)	$\mathbf{v}_m^0$ (m/s)	Solution Time (s)	$\Phi$
0	(-79, 90, -157)	(-10, -8, 0)	18.4	0.1
$(0, 5, 0)^T$	(-67, 55, -157)	(-9.4, -7.3, 2)	17.1	0.2
$(0, 10, 0)^T$	(-61, 35, -142)	(-9, -5, 2)	48.1	7965.1
$\mathbf{p}_m^0$ = initial mothership position; $\mathbf{v}_m^0$ = initial mothership velocity in North-East-Down frame.				

Figure 9 shows the optimal trajectory in the absence of wind. It can be seen that a horizontally flat drogue orbit (triangle-dot line) requires a horizontally flat mothership orbit (dash-dot line). The computed drogue trajectory (dashed line) follows the desired orbit precisely. The cable (solid line)

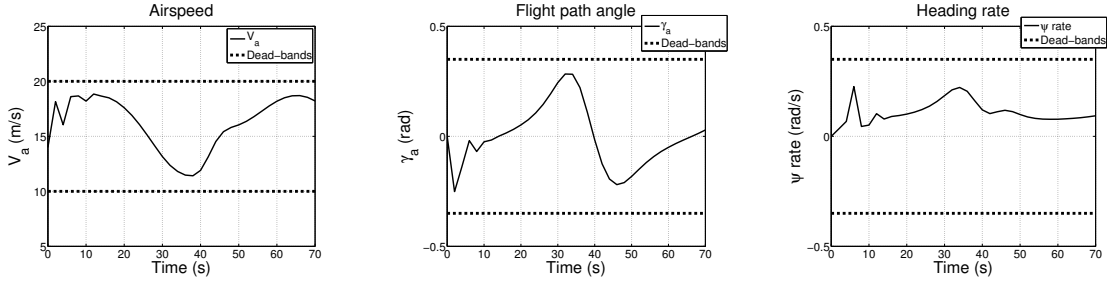


**Fig. 9 Optimal system trajectories in the absence of wind.**



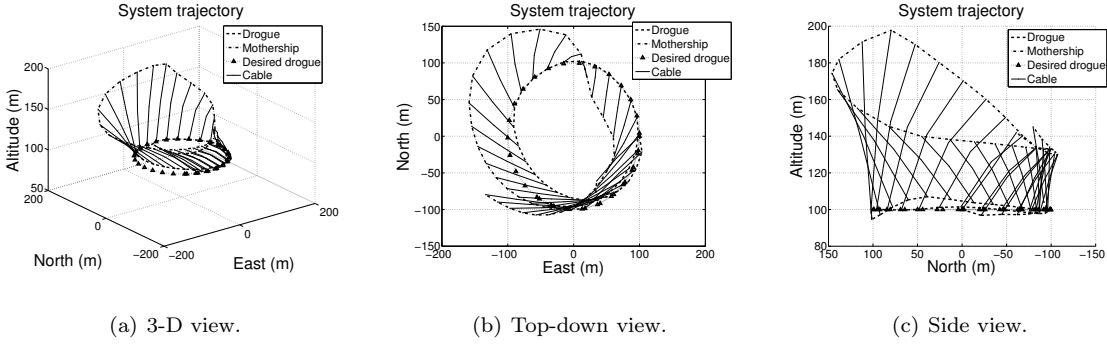
**Fig. 10 Optimal system trajectories using a desired drogue orbit with constant ground speed in the presence of 5 m/s wind from the west.**

is curved because of the aerodynamic forces exerted on the joint. Figure 10 shows the computed optimal system trajectory in the presence of 5 m/s wind from the west. It can be seen that the mothership orbit is inclined to produce a horizontally flat orbit of the drogue. The amplitude of the mothership’s altitude oscillation is approximately 40 m. Figure 11 shows the evolutions of the constraint variables  $V_a$ ,  $\gamma_a$  and  $\dot{\psi}$  of the mothership, where it can be seen (solid lines) that they remain within their limits (dashed lines). Figure 12 shows the optimal trajectory in the presence of 10 m/s wind from the west. It can be seen that the mothership orbit inclines more in a stronger wind. Because of the performance limits of the mothership, the resulting optimal orbit was unable to precisely place the drogue orbit onto the desired orbit, which results in a large value for the objective function in Table 2. The amplitude of the mothership’s altitude oscillation was approximately 70 m, while the amplitude of the drogue’s altitude oscillation was approximately 15 m. Figure 13 shows the evolutions of the constraints  $V_a$ ,  $\gamma_a$  and  $\dot{\psi}$  of the mothership. It can be seen that although all the constrained variables reach their limits during the flight, the optimizer was able to produce



(a) Airspeed  $V_a$  vs. time. (b) Flight path angle ( $\gamma_a$ ) vs. time. (c) Heading rate  $\dot{\psi}$  vs. time.

**Fig. 11** Evolution of constrained variables of the mothership using a desired drogue orbit with constant ground speed in the presence of 5 m/s wind from the west.



(a) 3-D view. (b) Top-down view. (c) Side view.

**Fig. 12** Optimal system trajectories using a desired drogue orbit with constant ground speed in the presence of 10 m/s wind from the west.

an optimal trajectory for the mothership to place the drogue orbit to essentially follow the desired orbit.

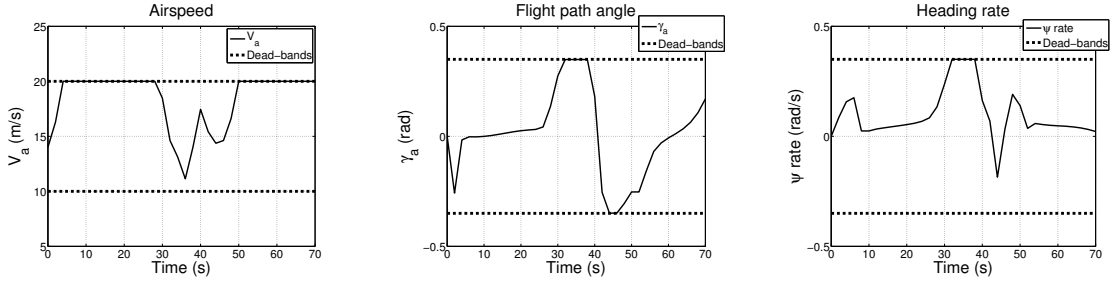
### B. Desired Drogue Orbit with Constant Airspeed

Because the autopilot on the MAV is typically designed to regulate a constant airspeed, in this section, a desired drogue trajectory with constant airspeed is used to calculate the desired mothership orbit. The time derivative of  $\mathbf{p}_{dr}^d(t)$  using Eqs. (13) to (15) is given by

$$\dot{\mathbf{p}}_{dr}^d = \begin{pmatrix} -r_{dr}^d \dot{\theta} \sin \theta \\ r_{dr}^d \dot{\theta} \cos \theta \\ 0 \end{pmatrix} = \mathbf{v}_{dr}^s + \mathbf{w}. \quad (16)$$

Thus, the airspeed of the drogue  $\|\mathbf{v}_{dr}^s\|$  is calculated as

$$\|\mathbf{v}_{dr}^s\| = \sqrt{\left(-r_{dr}^d \dot{\theta} \sin \theta - w_n\right)^2 + \left(r_{dr}^d \dot{\theta} \cos \theta - w_e\right)^2 + w_d^2}.$$



(a) Airspeed  $V_a$  vs. time.

(b) Flight path angle  $\gamma_\alpha$  vs. time.

(c) Heading rate  $\dot{\psi}$  vs. time.

**Fig. 13 Evolution of constrained variables of the mothership using a desired drogue orbit with constant ground speed in the presence of 10 m/s wind from the west.**

Given a desired drogue airspeed  $v_{dr}^a$ , it can be obtained by solving the quadratic equation

$$r_{dr}^2 \dot{\theta}^2 + 2r_{dr} (w_n \sin \theta - w_e \cos \theta) \dot{\theta} + w_n^2 + w_e^2 + w_d^2 - (v_{dr}^a)^2 = 0$$

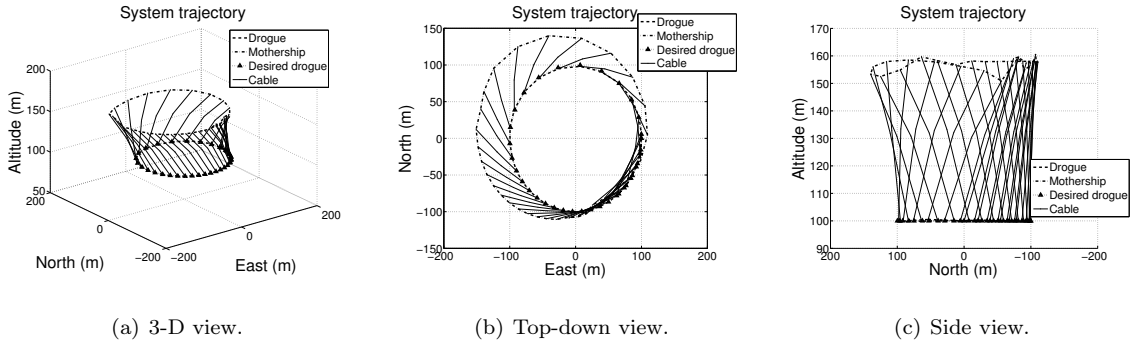
for  $\dot{\theta}$  resulting in the clockwise motion being given by

$$\dot{\theta} = \frac{w_e \cos \theta - w_n \sin \theta + \sqrt{(w_n \sin \theta - w_e \cos \theta)^2 - (w_n^2 + w_e^2 + w_d^2 - (v_{dr}^a)^2)}}{r_{dr}}. \quad (17)$$

The orbital period  $T_p^a$  can be calculated as [17]

$$T_p^a = \int_0^{2\pi} \frac{1}{\dot{\theta}} d\theta. \quad (18)$$

It is not difficult to see that  $T_p^a$  increases when the wind speed increases. In the presence of 5 m/s wind,  $T_p^a$  can be calculated as 60.51 s by using Eq. (18). When the wind increases to 10 m/s,  $T_p^a$  increases to 135.8 s. To guarantee that the optimal trajectory of the mothership has enough waypoints to produce an orbit, the starting and ending times are selected as  $t_0 = 0$  and  $t_1 = 70$  s for the case of 5 m/s wind, and  $t_1 = 150$  s for the case of 10 m/s wind. The initial configuration and solution results are shown in Table 3. It can be seen that when the wind speed increased to 10 m/s, the objective function value increased to 755.2. This is because the existing performance limits of the mothership made the resulting optimal orbit unable to precisely place the drogue orbit onto its desired value. The parameters of the desired drogue orbit were selected as  $r_{dr} = 100$  m and  $v_{dr}^a = 12$  m/s to compare the results with those in the previous section.



**Fig. 14 Optimal system trajectories using a desired drogue orbit with constant airspeed in the presence of 5 m/s wind from the west.**

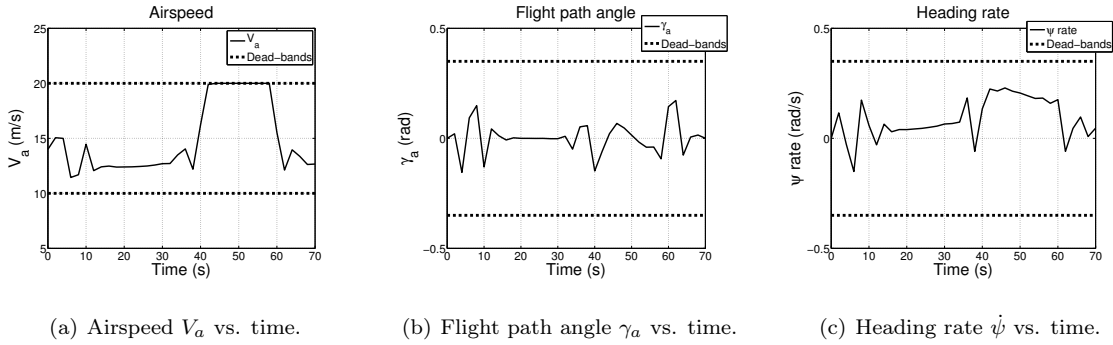
**Table 3 Solution results using the desired drogue orbit with constant airspeed.**

Wind speed (m/s)	$\mathbf{p}_m^0$ (m)	$\mathbf{v}_m^0$ (m/s)	Solution Time (s)	$\Phi$
$(0, 5, 0)^T$	$(-74, 63, -159)$	$(-6, -9, 0)$	18.8	17.9
$(0, 10, 0)^T$	$(-20, 40, -159)$	$(-3, -4, 0)$	39.8	755.2

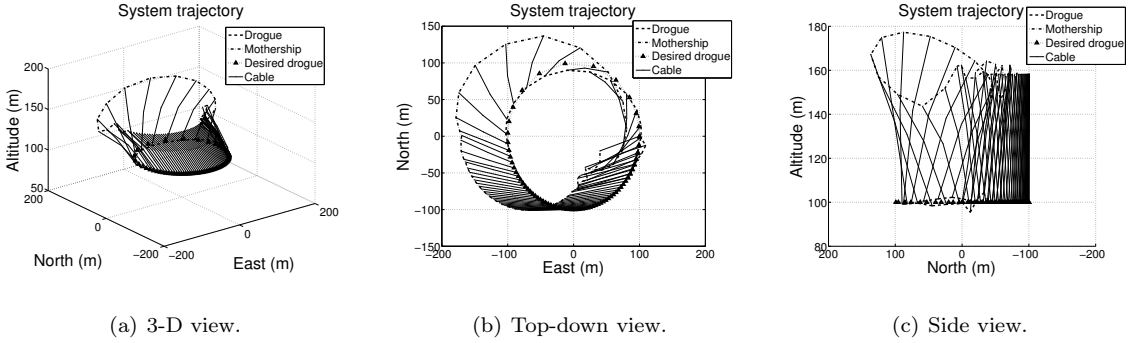
$\mathbf{p}_m^0$  = initial mothership position;  $\mathbf{v}_m^0$  = initial mothership velocity in North-East-Down frame.

Figure 14 shows the optimal trajectory in the presence of 5 m/s wind from the west. It can be seen that the offset of the mothership orbit center in Fig. 14 (b) is smaller than in Fig. 10 (b). It also can be seen that the inclination of the mothership orbit in Fig. 14 (c) was much smaller than in Fig. 10 (c). The amplitude of the mothership's altitude oscillation was approximately 10 m. Because the desired airspeed of the drogue was constant, it can be seen that the waypoints placed close together when the system is flying upwind (west), and sparsely when the system was flying downwind (east). Figure 15 shows the evolutions of the constrained variables  $V_a$ ,  $\gamma_a$  and  $\dot{\psi}$  of the mothership. It can be seen that the desired airspeed of the mothership in Fig. 15 (a) reached its upper limit, while in the same wind condition, the desired airspeed in Fig. 11 (a) was still within the limits.

Figure 16 shows the optimal trajectory in the presence of 10 m/s wind from the west. It can be seen that the offset of the mothership orbit center and the inclination of the desired mothership orbit become larger than those in Fig. 14. The actual drogue orbit shows both horizontal and vertical offsets, which implies that the constrained variables of the mothership reach their limits during the



**Fig. 15** Evolution of constrained variables of the mothership using a desired drogue orbit with constant airspeed in the presence of 5 m/s wind from the west.



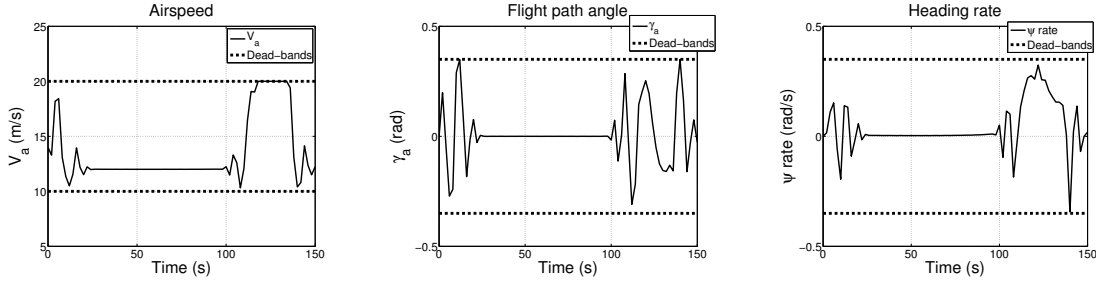
**Fig. 16** Optimal system trajectories using a desired drogue orbit with constant airspeed in the presence of 10 m/s wind from the west.

flight. The mothership trajectory in Fig. 16 (c) may look not look smooth, but this is an artifact of quantizing the trajectory waypoints for display purposes. Figure 17 shows the evolutions of the constrained variables  $V_a$ ,  $\gamma_a$  and  $\dot{\psi}$  of the mothership. It can be seen that the desired airspeed of the mothership reached the limit, while  $\gamma_a$  and  $\dot{\psi}$  still remained within their limits.

### C. Transitions between Straight Level Flight and Orbital Flight

The transition between straight-and-level flight and orbital flight (and vice versa) needs special attention for a towed cable system because the cable may become slack when the mothership turns and may experience large and sudden forces that may break the cable [15]. To prevent the tension forces exerted on the cable from exceeding the loading limit, an optimal trajectory of the mothership is needed to keep the tension forces within their limits during the transition. In this section, the focus is on the tow-in motion in which the system flies from a straight flight into an orbit. The





(a) Airspeed  $V_a$  vs. time.

(b) Flight path angle  $\gamma_a$  vs. time.

(c) Heading rate  $\dot{\psi}$  vs. time.

**Fig. 17 Evolution of constrained variables of the mothership using a desired drogue orbit with constant airspeed in the presence of 10 m/s wind from the west.**

tension forces on the cable are selected as additional constraints in the optimization algorithm with the limits  $\|\mathbf{T}_i\| \in [0, 10]$  N,  $i = 1, 2$ . Optimal results in different wind conditions are presented in this section.

Letting  $t_a$  be the time when transition starts, the desired drogue trajectory in a tow-in motion can be written as

- (1) when  $t \in [0, t_a]$ , the straight-line trajectory of the drogue is given by

$$p_{dr_n}^d(t) = -v_{dr}^g t$$

$$p_{dr_e}^d(t) = r_{dr}^d$$

$$p_{dr_d}^d(t) = -h_0;$$

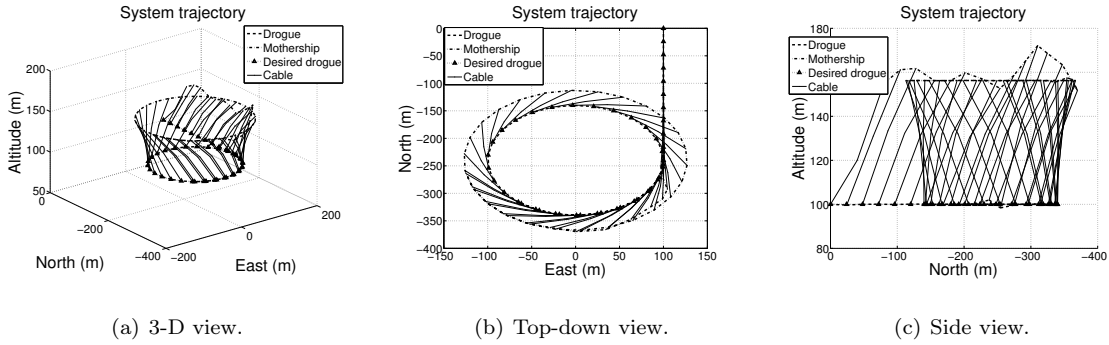
- (2) when  $t \in (t_a, t_1]$ , the circular trajectory of the drogue is given by

$$p_{dr_n}^d(t) = -v_{dr}^g t_a + r_{dr}^d \cos \theta(t)$$

$$p_{dr_e}^d(t) = r_{dr}^d \sin \theta(t)$$

$$p_{dr_d}^d(t) = -h_0.$$

In this section, the desired circular drogue orbit with constant ground speed is used, i.e.,  $\theta(t) = v_{dr}^g t / r_{dr}^d$ . The parameters are selected as  $r_{dr} = 100$  m,  $v_{dr}^g = 12$  m/s,  $t_a = 20$  s and  $t_1 = 80$  s in the optimization algorithm. The initial configuration and solution results are shown in Table 4. When the wind increased to 10 m/s, the objective function increased to 1751. This is because the existing performance limits of the mothership made the resulting optimal orbit unable to precisely place the drogue orbit onto the desired orbit.



**Fig. 18 Optimal system trajectories in the transitional flight in the absence of wind.**

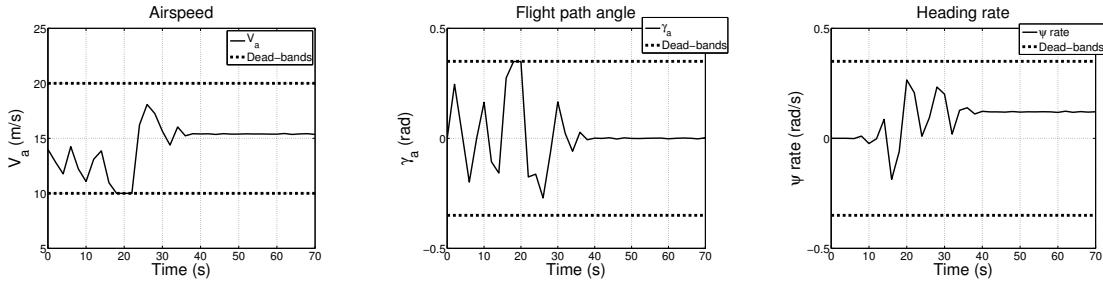
**Table 4 Solution results of transitional flight in different wind conditions.**

Wind speed (m/s)	$\mathbf{p}_m^0$ (m)	$\mathbf{v}_m^0$ (m/s)	Solution Time (s)	$\Phi$
0	$(-80, 100, -155)$	$(-14, 0, 0)$	23.2	20.3
$(0, 5, 0)^T$	$(-80, 67, -155)$	$(-14, 0, 0)$	20.1	21.6
$(0, 10, 0)^T$	$(-70, 41, -143)$	$(-14, 0, 0)$	29.7	1751.0

$\mathbf{p}_m^0$  = initial mothership position;  $\mathbf{v}_m^0$  = initial mothership velocity in North-East-Down frame.

Figure 18 shows the optimal trajectory during a tow-in maneuver in the absence of wind. It can be seen that it takes one quarter circle for the mothership to complete the transition. The mothership trajectory had an altitude oscillation of approximately 20 m during the transition. Figure 19 shows the evolutions of the constrained variables  $V_a$ ,  $\gamma_a$  and  $\dot{\psi}$  of the mothership. It can be seen that the airspeed reached its lower limits during the transition. The large oscillation of  $\gamma_a$  explained the altitude oscillation of the mothership during the transition. Figure 20 shows the evolution of the tension forces on the cable in the transitional flight. The tension forces had a small oscillation (approximately 1 N), while remaining within their limits.

Figure 21 shows the optimal trajectory during a tow-in maneuver in the presence of 5 m/s wind from West. It can be seen that the mothership trajectory had an altitude oscillation of approximately 15 m during the transition. Figure 22 shows the evolutions of the constrained variables  $V_a$ ,  $\gamma_a$  and  $\dot{\psi}$  of the mothership. It can be seen that they remain within the specified limits during the transition. Figure 23 shows that the tension forces have an oscillation of approximately 2 N during the transition while remaining within the specified limits.

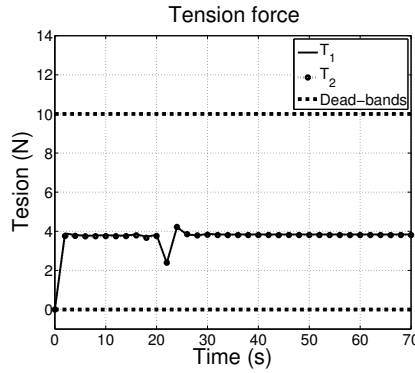


(a) Airspeed  $V_a$  vs. time.

(b) Flight path angle  $\gamma_a$  vs. time.

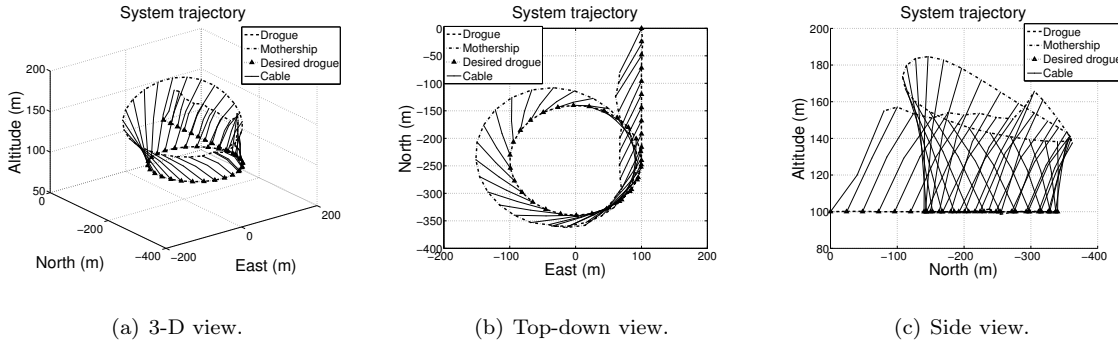
(c) Heading rate  $\dot{\psi}$  vs. time.

**Fig. 19 Evolution of constrained variables of the mothership in the transitional flight in the absence of wind.**

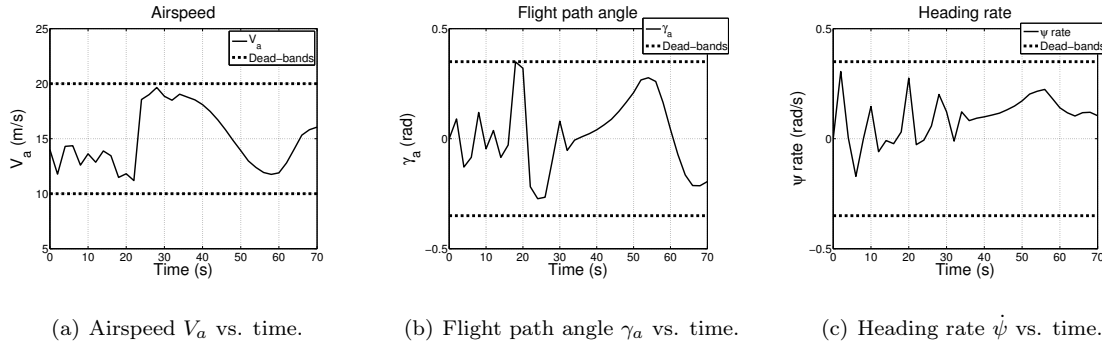


**Fig. 20 Evolution of the tension forces of the cable in the transitional flight in the absence of wind.**

Figure 24 shows the optimal trajectory during a tow-in maneuver in the presence of 10 m/s wind from West. The mothership trajectory has an altitude oscillation of approximately 10 m during the transition. Figure 25 shows that the airspeed of the mothership  $V_a$ ,  $\gamma_a$  reaches its upper bound, while  $\gamma_a$  and  $\dot{\psi}$  remain within their limits during the transition. Therefore, the resulting optimal orbit was unable to precisely track the desired orbit, and this explains the large objective function value of 1751 in Table 4. Figure 26 shows that the cable tension had an increase (approximately 3 N), while they remained within the limits during the transition. The altitude oscillation of the mothership increases as the wind speed increases and the tension forces have small oscillations during the transition and remain within their limits.



**Fig. 21 Optimal system trajectories in the transitional flight in the presence of 5 m/s wind from West.**

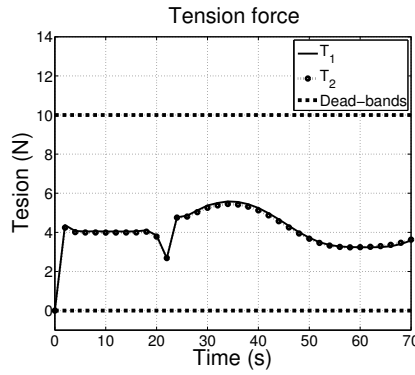


**Fig. 22 Evolution of constrained variables of the mothership in the transitional flight in the presence of 5 m/s wind from West.**

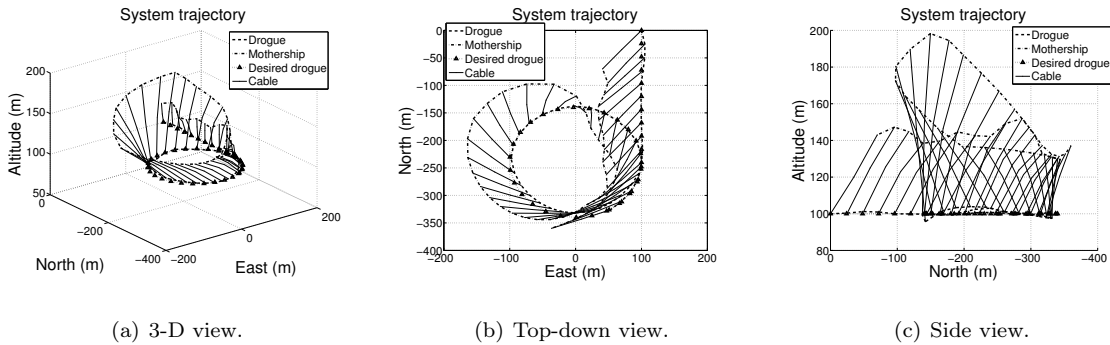
## VI. Flight test results

Flight tests were conducted to validate the solutions obtained in the previous section. The desired trajectories of the mothership were pre-calculated off-line using different wind speeds by assuming that the direction of the wind comes from south. The required trajectory of the mothership was updated after each orbit according to the average estimated wind direction and magnitude.

Unfortunately the experimental results were not very satisfying in that the discrepancy between the desired drogue position and the actual drogue position was sometimes quite large. There are several possible explanations. First, the computed trajectories are open-loop trajectories computed at a small number of wind conditions. As the orbit trajectories were flown, the wind was averaged over the entire orbit, and a new trajectory is selected from the database at the beginning of each orbit. Wind variation throughout the orbit will obviously have a big impact. It may also be the case that actual flight trajectories are sensitive to incorrect parameters like elasticity and drag.



**Fig. 23** Evolution of the tension forces of the cable in the transitional flight in the presence of 5 m/s wind from West.

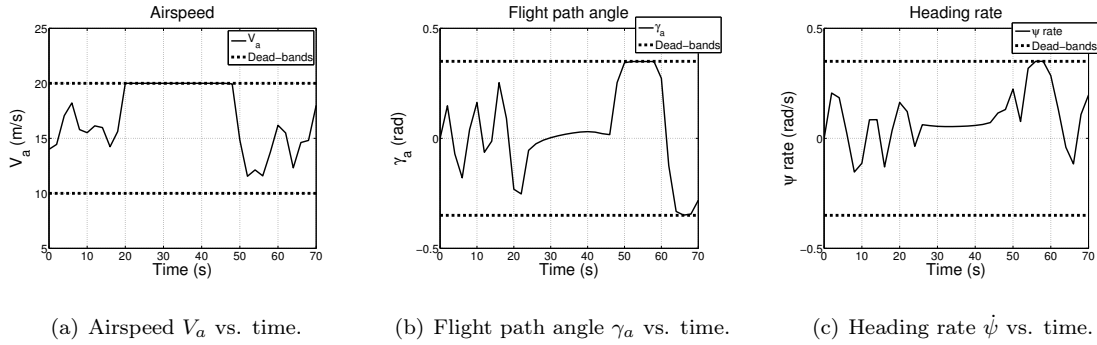


**Fig. 24** Optimal system trajectories in the transitional flight in the presence of 10 m/s wind from West.

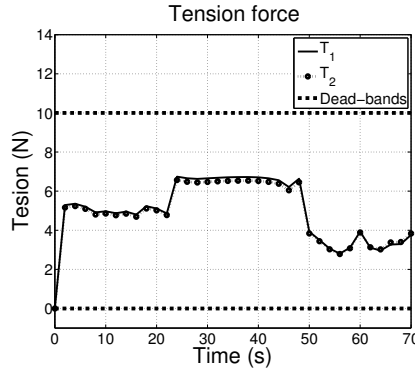
In addition, after the drogue experienced disturbances, the configuration of the mothership-cable-drogue system no longer matched conditions in the database and the computed trajectories were no longer valid. Flight implementation of the methods described in this paper will require the computation and storage of a large number of initial conditions in various wind conditions, and more frequent switches between trajectories during the flight.

## VII. Conclusion

This paper presents a strategy for generating optimal trajectories for the constrained towing vehicle (mothership) of an aerially towed cable system using model predictive control (MPC). To select an appropriate number of cable links in the optimization, model validation was conducted by comparing the flight test data with the results from the simulation with different numbers of cable link. The results indicate that a different number of cable links (1, 2, 5, 20, 30) did not result



**Fig. 25** Evolution of constrained variables of the mothership in the transitional flight in the presence of 10 m/s wind from West.



**Fig. 26** Evolution of the tension forces of the cable in the transitional flight in the presence of 10 m/s wind from West.

in a significant difference in the resulting motion. Two cable links were chosen as a compromise between accuracy and model complexity. The optimization formulation using model predictive control was presented by employing a combination of the squared-error and  $L_1$ -norm objective function. Different desired drogue paths were employed to examine the strategy of the optimal trajectory generation. For the desired drogue orbit with constant ground speed, stronger wind required larger maneuvers on both the airspeed and flight path angle of the mothership, while for the desired drogue orbits with constant airspeed, stronger wind required larger maneuvers only on the airspeed of the mothership. In the transitional flight, as the wind increased, the altitude oscillation of the mothership during the transition decreased. The tension forces on the cable were also kept within the limits during the transition. This MPC-based optimal trajectory generation strategy can be a framework for specifying any arbitrary flight path of the towed body by optimizing the action

of the towing vehicle subject to constraints and wind disturbance. Practical implementation will require that a large number of optimal trajectories be computed from different initial conditions and stored in a database, and that the resulting trajectories be stitched together during flight based on actual flight conditions.

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