# MINLP with Combined Interior Point and Active Set Methods

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# Presentation Overview

- NLP Benchmarking
  - Hock-Schittkowski
  - Dynamic optimization
  - Biological models
- Combining Interior Point and Active Set
- MINLP Benchmarking
  - MacMINLP
  - MINLP Model Predictive Control
  - Chiller Thermal Energy Storage
  - Unmanned Aerial Systems
- Future Developments

# Overview of Benchmark Testing

- NLP Benchmark Testing
  - APOPT[1], BPOPT[1], IPOPT[2], SNOPT[3], MINOS[3]
  - Problem characteristics:
    - Hock Schittkowski, Dynamic Opt, SBML
    - Nonlinear Programming (NLP)
    - Differential Algebraic Equations (DAEs)
    - APMonitor Modeling Language

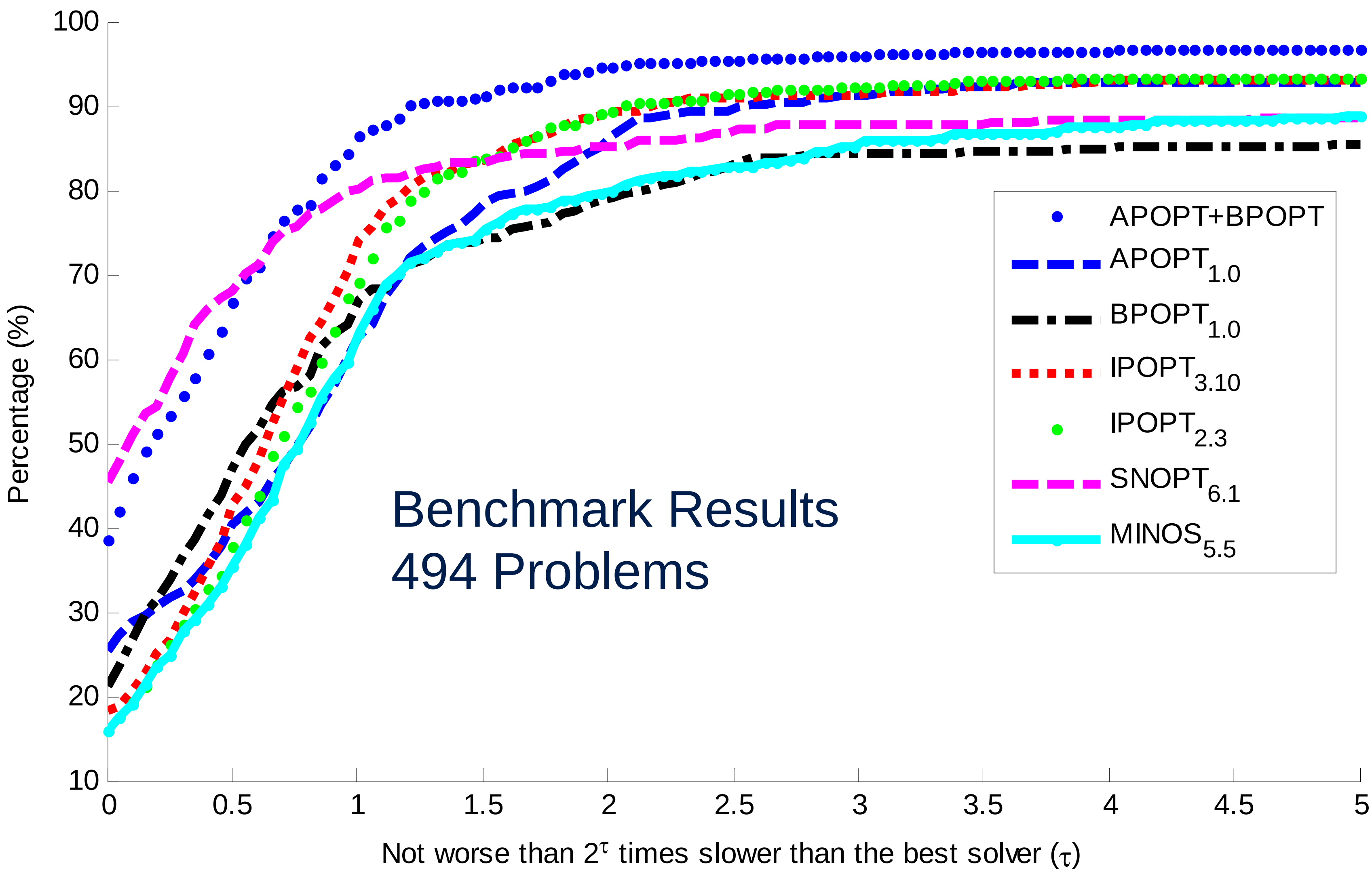
$$\begin{aligned} &\min J(x,y,u) \\ &s.t.\ 0 = f\left(\frac{\partial x}{\partial t}, x, y, u\right) \\ &\quad 0 = g(x,y,u) \\ &\quad 0 < h(x,y,u) \\ &\quad x, y \in \Re^n \ u \in \Re^m \end{aligned}$$

- MINLP Benchmark Testing
  - APOPT[1], BPOPT[1], BONMIN[2]
  - Problem characteristics:
    - MacMINLP, Industrial Test Set
    - Mixed Integer Nonlinear Programming (MINLP)
    - Mixed Integer Differential Algebraic Equations (MIDAEs)
    - APMonitor & AMPL Modeling Language

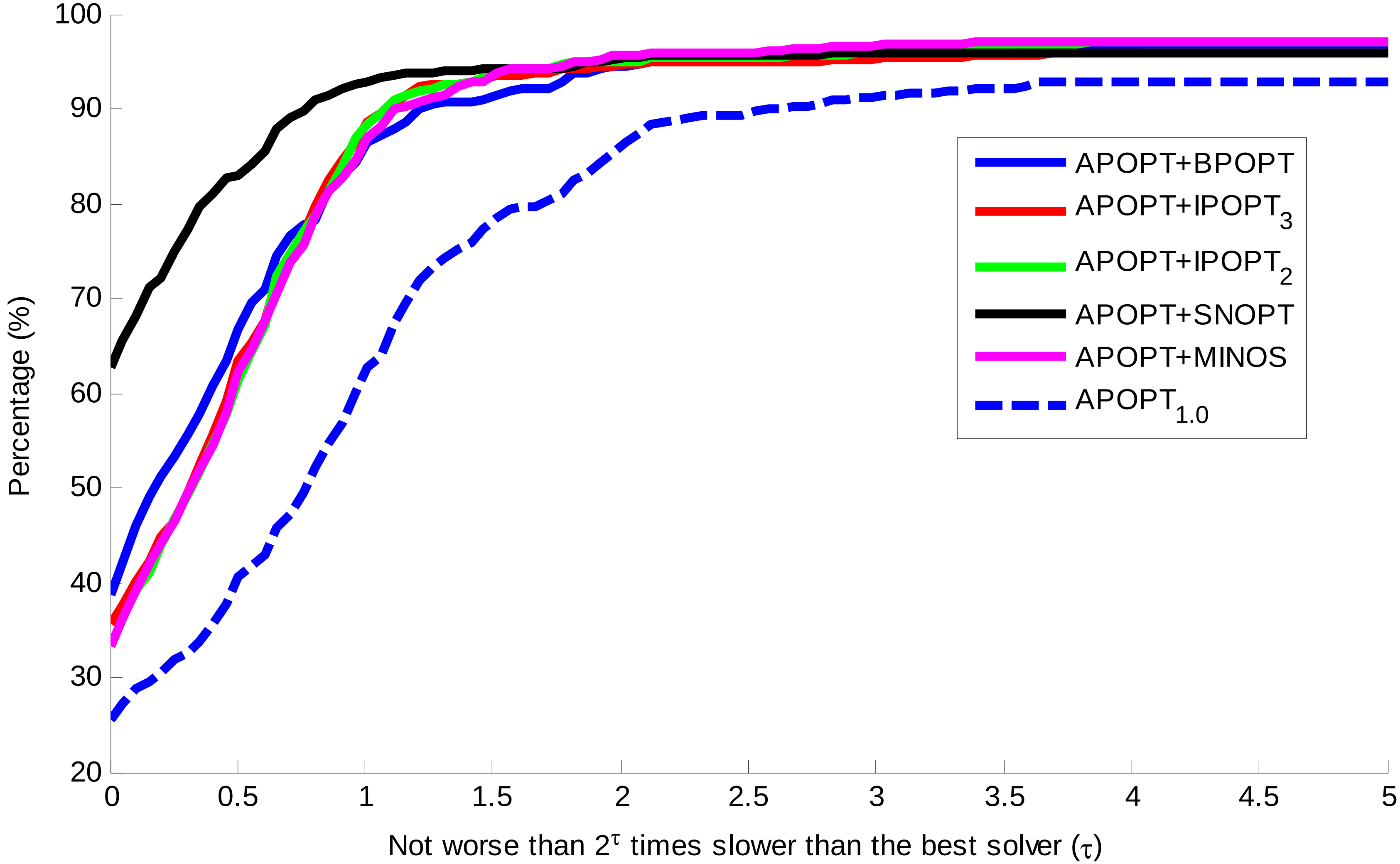
$$\begin{aligned} &\min J(x,y,u,z) \\ &s.t.\ 0 = f\left(\frac{\partial x}{\partial t}, x, y, u, z\right) \\ &\quad 0 = g(x,y,u,z) \\ &\quad 0 < h(x,y,u,z) \\ &\quad x, y \in \Re^n \ u \in \Re^m \ z \in \mathrm{I}^m \end{aligned}$$

1–APS, LLC 2–EPL, 3–SBS, Inc.

# NLP Benchmark – Summary (494)



# Complementary Solvers



# MacMINLP Benchmark Summary Results

| Problem Name | Platform | Variables | Integers | Equations | NLP Obj | MINLP Obj | Solution Time | Status |
|---|---|---|---|---|---|---|---|---|
| an_integer_test | APM | 10 | 4 | 4 | 1.55E+02 | 1.74E+02 | 0.031 | Success |
| batch | APM | 107 | 24 | 73 | 2.59E+05 | 2.86E+05 | 0.25 | Success |
| c-reload-14a | APM | 510 | 168 | 308 | -1.01E+00 | -1.01E+00 | 8.865 | Success |
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| c-reload-14c | APM | 510 | 168 | 308 | -9.91E-01 | -9.56E-01 | 48.344 | Failure |
| c-reload-14d | APM | 510 | 168 | 308 | -1.03E+00 | -1.03E+00 | 6.724 | Success |
| c-reload-14e | APM | 510 | 168 | 308 | -1.03E+00 | -1.03E+00 | 4.852 | Success |
| c-reload-14f | APM | 510 | 168 | 308 | -1.01E+00 | -1.01E+00 | 6.614 | Success |
| geartrain1 | APM | 4 | 4 | 0 | 6.30E-13 | 7.78E-07 | 0.016 | Success |
| mittelman | APM | 27 | 16 | 7 | 5.02E+00 | 1.30E+01 | 0.094 | Success |
| mittelman | AMPL | 16 | 16 | 7 | 5.02E+00 | 1.30E+01 | 0.463 | Success |
| optprloc | AMPL | | | | -1.64E+01 | -8.05E+00 | 15.682 | Success |
| spring | AMPL | 17 | 12 | 8 | | | 0.31 | Success with IPOPT pre-solver |
| space-25 | AMPL | 893 | 750 | 235 | 4.84E+02 | 4.88E+02 | | Success |
| space-960 | AMPL | 15137 | 9600 | 8417 | 7.61E+06 | | | Timeout - DOF exceeded |
| stockcycle-1 | AMPL | 480 | 480 | 97 | 1.18E+05 | 2.49E+05 | 345.2 | Success |
| stockcycle-1 | AMPL | 480 | 480 | 97 | 1.18E+05 | 1.43E+05 | 11.584 | Success |
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# Case Study- MPC with MINLP

- PCT40 is a basic process control unit produced by Armfield
- Main Devices: Large Process Vessel, CSTR, Hot water Tank
- Sensors: temperature, pressure, and level sensors
- Devices: valves, heating coils, and pumps

*How does this work for MINLP Problems?*

Objective: Keep the tank level at 100 mm

Continuous: Proportional Solenoid Valve (PSV)

Discrete: On/Off Valve (SOL1)

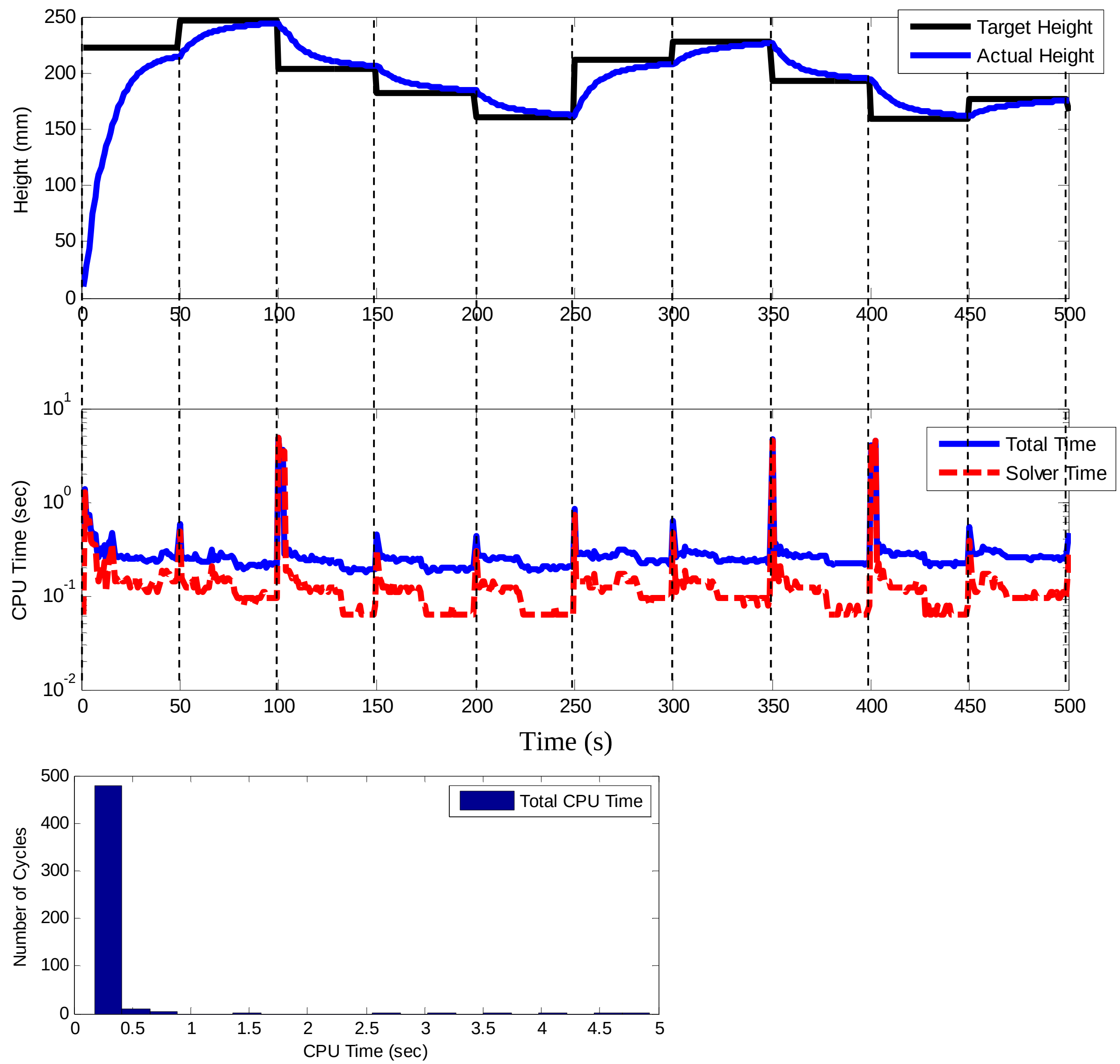
# MINLP Model Predictive Control

<u>MIDAE System</u>
Variables: 576
Integers: 16
Equations: 544

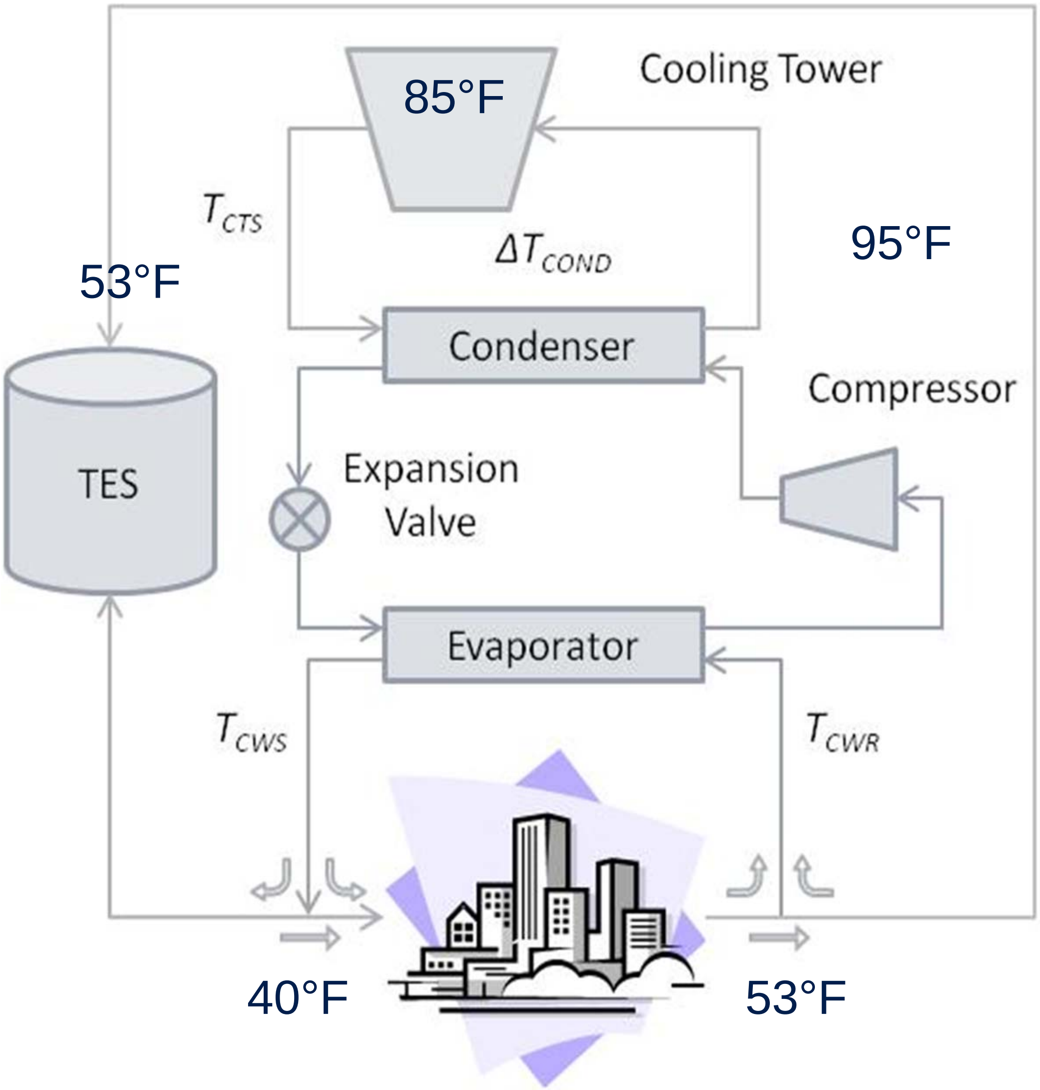
<u>Features</u>
Warm Start Solutions

Discrete (16) and Continuous (16) Decisions

CPU Increases with Setpoint Changes



# Chiller Thermal Energy Storage




<u>MINLP System</u>
Variables: 672
Integers: 24
Equations: 576

<u>Features</u>
Cold Start Solution

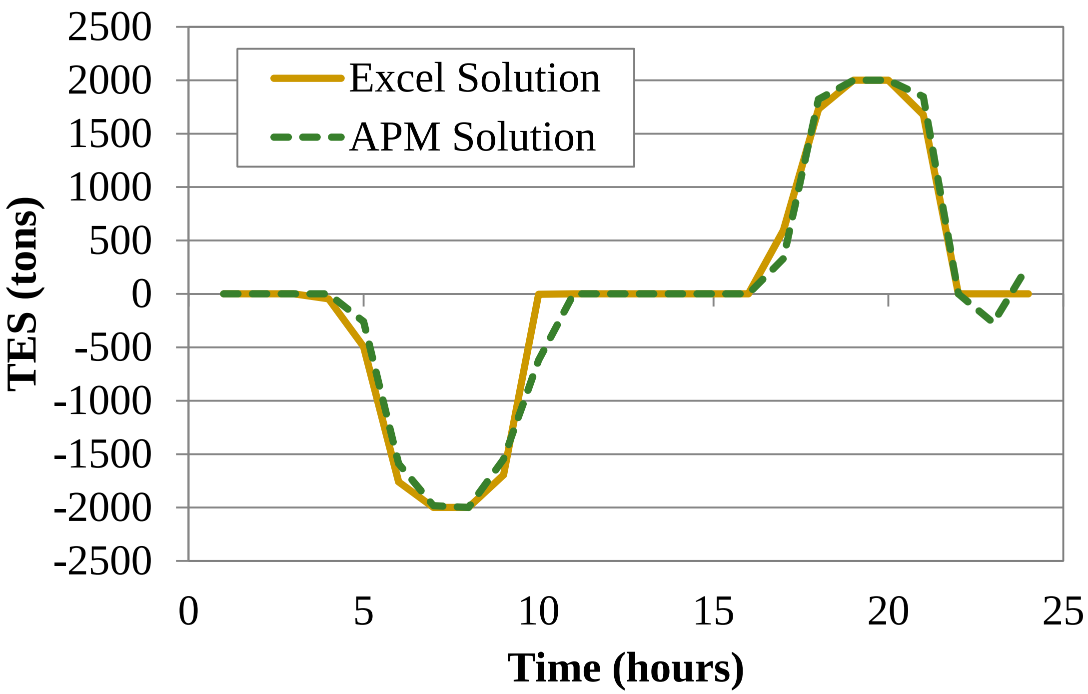
Discrete (24) and Continuous (24) Decisions

Contributed by Wesley Cole,
University of Texas at Austin

# Results

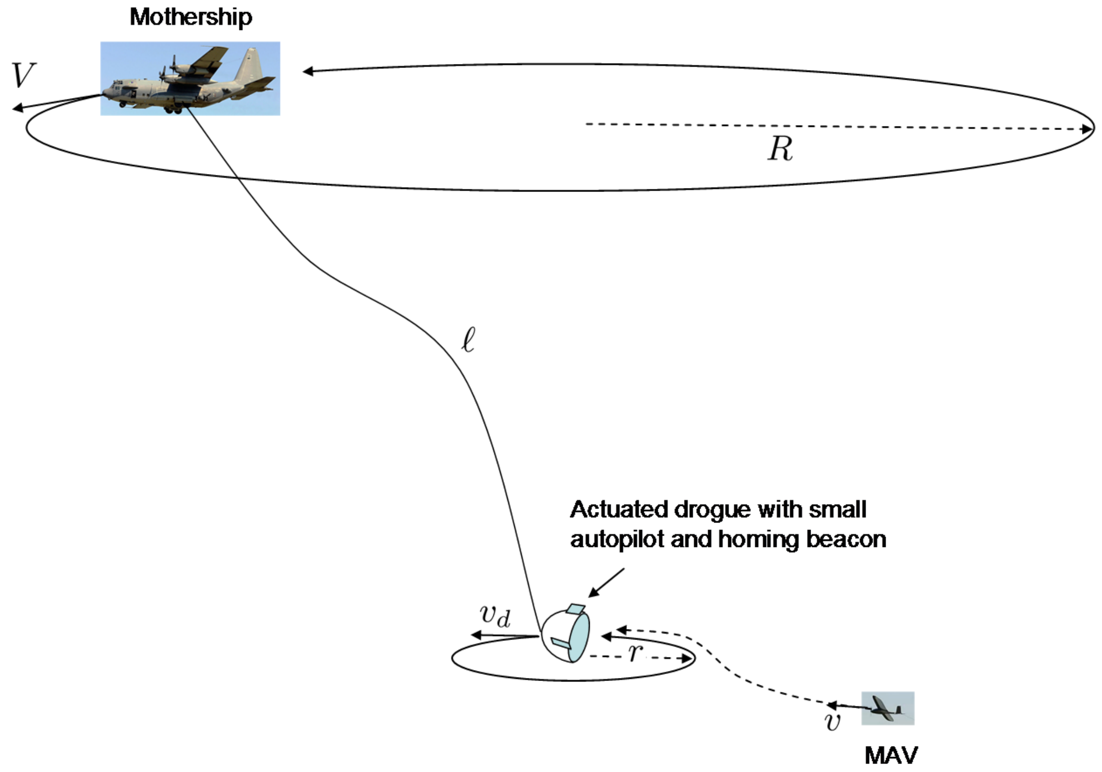
| | BONMIN | APOPT | Excel | MATLAB |
|---|---|---|---|---|
| **Cost ($/day)** | $ 807.53 | $ 804.65 | $ 801.93* | $ 802.13* |
| **CPU Time (sec)** | 11.6 | 45.0 | 58.4* | 0.1* |

* = Relaxed Formulation (NLP)



# Unmanned Aerial Systems Estimation Problem

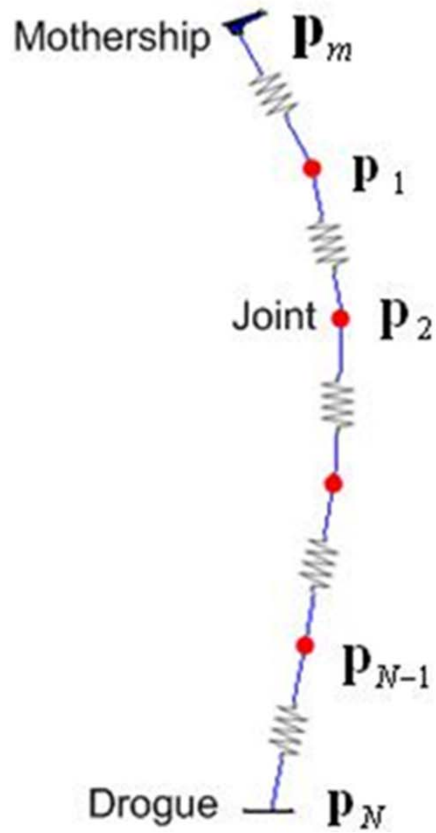
Decision Variables
- Number of Cable Links
- Drag Coefficient
- Cable Stretching
- Wind Disturbance



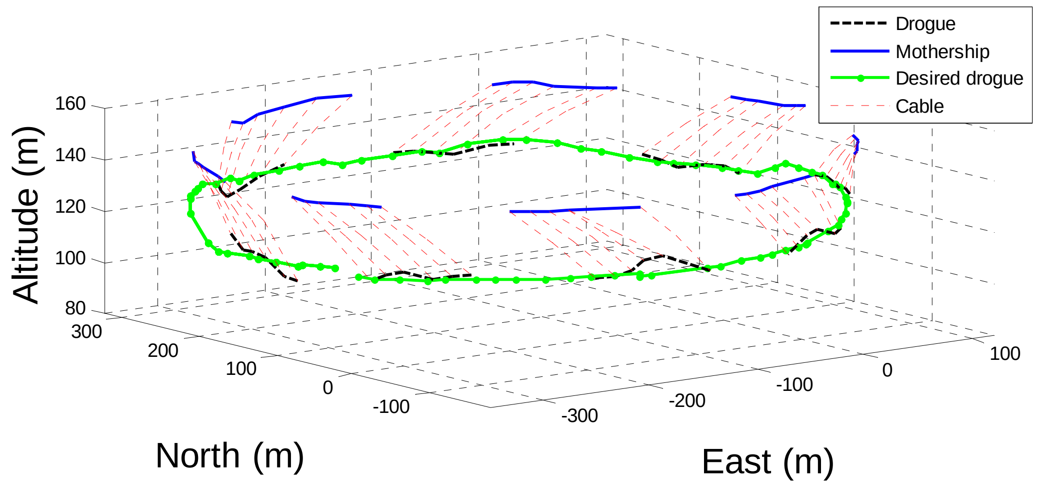
# UAS System Dynamics

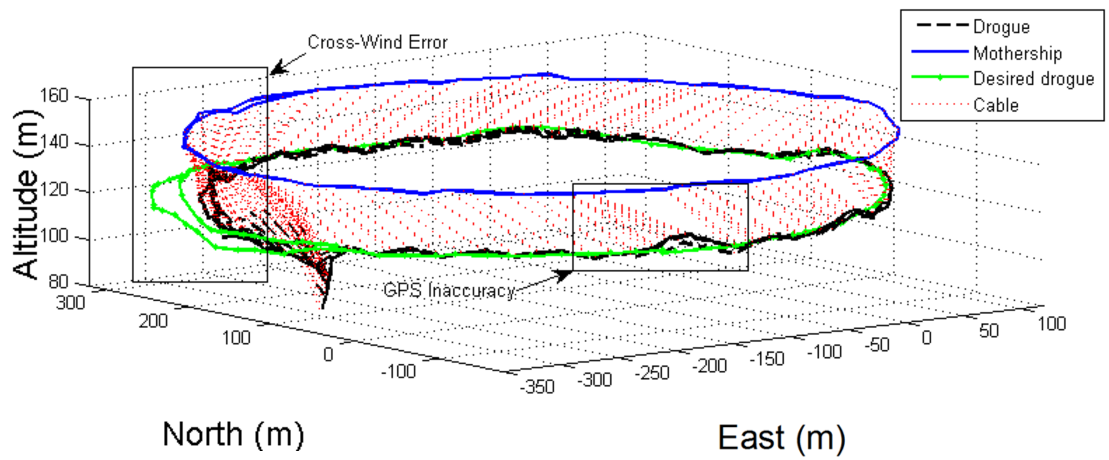
- Cable-drogue dynamics using Newton 2nd law

$$
\begin{aligned}
\min_{\mathbf{u}(t_0,t_1)} \Phi \;=\; & \mathbf{w}_{hi}^T \mathbf{e}_{hi} + \mathbf{w}_{lo}^T \mathbf{e}_{lo} + \mathbf{y}^T \mathbf{c}_y + \mathbf{u}^T \mathbf{c}_u \\
& + \int_{t_0}^{t_1} \left(\mathbf{p}_{dr}(\delta) - \mathbf{p}_{dr}^{d}(\delta)\right)^T \left(\mathbf{p}_{dr}(\delta) - \mathbf{p}_{dr}^{d}(\delta)\right) d\delta \\
\text{s.t. } & \mathbf{f}\left(\dot{\mathbf{x}}, \mathbf{x}, \mathbf{y}, \mathbf{u}, \mathbf{d}\right) = \mathbf{0} \\
& \mathbf{g}_{iec}\left(\dot{\mathbf{x}}, \mathbf{x}, \mathbf{y}, \mathbf{u}, \mathbf{d}\right) \geqslant \mathbf{0} \\
& \tau \frac{\partial \mathbf{y}_{hi}}{\partial t} + \mathbf{y}_{hi} = \mathbf{sp}_{hi} \\
& \tau \frac{\partial \mathbf{y}_{lo}}{\partial t} + \mathbf{y}_{lo} = \mathbf{sp}_{lo}.
\end{aligned}
$$



# MINLP Flight Estimation




Number of state variables: 384
Number of total equations: 380
Solve time: 0.1-0.5 seconds

# Conclusions

- Solver Interfaces for APOPT
  - APMonitor Modeling Language (http://apmonitor.com)
  - AMPL Modeling Language (http://ampl.com)
- Current Developments
  - Mixed Integer Nonlinear Programming (MINLP)
  - MINLPs from Discretized Dynamic Systems
  - Combine strengths of Active Set (APOPT) and Interior Point (IPOPT) methods
    - APOPT: Better for restart from nearby solution
    - APOPT: Better for identification of infeasible solutions
    - IPOPT: Better for initialization
    - IPOPT: Better for large degrees of freedom

# Future Development Plans

- Active Development Efforts
  - Mixed Integer solvers that exploit problem structure
  - Interfaces to other scripting languages
- BPOPT MINLP solver development
  - Interior Point Methods to MINLP problems
  - Automated Solver Parameter Tuning
- Additional Benchmarking
  - COPS, etc.
- New APOPT Interfaces
  - Python
  - MATLAB
- Industrial and Academic Collaborators

# Backup Slides

- What are the Best Complementary Solvers?

# What are the Best Complementary Solvers?