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Motion Planning and Control for Mothership-cable-drogue Systems in Aerial Recovery of Micro Air Vehicles

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***Abstract*— Aerial recovery of micro air vehicles (MAVs) presents a challenging problem in multi-vehicle dynamics and control. This paper presents a method for recovering MAVs in flight using a mothership and towed drogue, in which the mothership executes an orbit that places the drogue in a stable, slower orbit that can be tracked by a MAV. The differential flatness property of the system is exploited to calculate mothership trajectories from desired drogue orbits, and a backstepping controller is proposed that enables accurate mothership trajectory tracking. Simulation results based on multi-link cable-drogue systems verify the feasibility and robustness of the approach.**

I. INTRODUCTION

In recent years, the use of unmanned air systems (UASs) has increased dramatically in both military and civilian fields, with applications ranging from intelligence, surveillance, and reconnaissance (ISR) to wilderness search and rescue. Micro air vehicles (MAVs), with wingspans typically less than 15 inches, have been increasingly used and have the potential to open new application areas and broaden the availability of UAS technology.

The ability to deploy MAVs locally and in large numbers opens many opportunities, but recovery of MAVs is problematic in certain scenarios [1]. For instance, if a soldier deploys a backpackable MAV on the battlefield to gather time-critical ISR information, it is undesirable for the MAV to return to the soldier because this could disclose his/her location to the enemy. Additionally, if a large mothership deploys multiple MAVs in a remote location for ISR, wildfire monitoring, or other surveillance, the MAVs may not have sufficient range to return home. Similarly, in disaster areas that are too remote or dangerous, MAV search or monitoring platforms may not be recovered by ground personnel.

One solution to this problem is to use a mothership as an aerial recovery platform for MAVs. The primary challenge with this approach is the high speed of the mothership relative to the MAV, which makes direct MAV/mothership rendezvous and capture impractical. Furthermore, aerial recovery must be highly accurate, as the rendezvous and capture must be coordinated in both time and space. The approach taken in the present work is to employ a capture device (drogue) that is towed by a larger mothership, as shown in Figure 1. In this method, the mothership enters an orbit designed to cause the towed drogue to execute an orbit of smaller radius and lower speed (less than the nominal speed of the MAV). The MAV then enters the drogue orbit at its nominal airspeed and overtakes the drogue with a relatively slow closing speed. In the terminal stages of rendezvous and capture, a vision-based homing algorithm, such as proportional navigation (PRONAV), is used to close the gap between MAV and drogue.

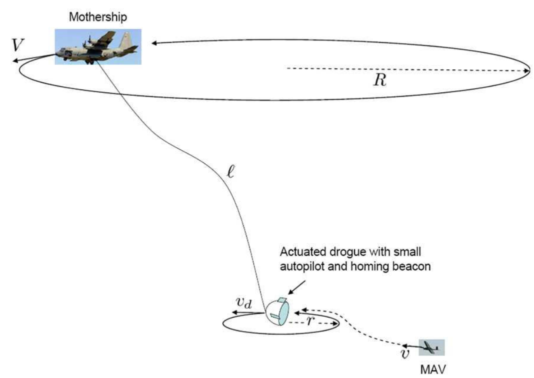

Fig. 1. Basic aerial recovery concept. The mothership recovers a MAV by towing a drogue that is actuated and can maneuver and communicate with the MAV to facilitate successful capture.

For the case of passive drogues, which can only be controlled indirectly via the mothership, it is necessary to have a method to calculate the orbit that the mothership must execute to place the drogue in an orbit suitable for aerial recovery of a MAV. We assume that the drogue is passive, i.e., it is only controllable indirectly via the mothership and cable, and that it is instrumented with a small autopilot and therefore has access to its own acceleration, angular rates, airspeed, and GPS location. Under these assumptions, the basic idea is to control the motion of the mothership so that the drogue enters a specified stable orbit whose radius r is greater than the minimum turning radius of the MAV, at an airspeed that is slightly below the nominal airspeed of the MAV.

In recent decades, motion control strategies to address related problems have appeared in the literature. The concept of differential flatness of the system is exploited in [2] to plan towplane paths that minimize the motion of the drogue. This work shows that the trajectory of the towplane is uniquely prescribed by the motion of the drogue. Unfortunately,

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the algorithm as presented in [2] has numerical stability issues. In [3], sequential quadratic programming is used to plan open-loop trajectories for the towplane. Reference [4] addresses the problem of entering and exiting the orbit with the cable deployed, and open-loop strategies are derived that minimize the tension on the cable and the drogue. Reference [4] also addresses the problem of deploying the cable from the towplane using a winch after the towplane is in its orbit. The majority of prior work in this area is related to the dynamics and stability of the drogue; few studies have explored specific strategies for accurate control of the mothership-cable-drogue system.

Once the desired trajectory of the mothership is calculated using methods from the previous section, a Lyapunov-based backstepping approach [5] can be used to find the control inputs that cause the mothership to track the trajectory. This is inspired, in part, by the work of [6] and [7]. Reference [6] proposes an output maneuvering controller for a class of strict feedback nonlinear processes and applies it to path-following for fully actuated ships. Reference [7] combine adaptive switching supervisory control with a nonlinear Lyapunov-based (backstepping) tracking control law for underactuated autonomous vehicles. However, the foci of [6] and [7] are primarily on low speed ground and water vehicles instead of high speed aerial vehicles which are characterized by more complex dynamics and demand faster realtime feedback control.

In this paper, we focus on the motion planning and control of the mothership, while we will briefly describe the dynamic model of the cable-drogue system, and our emphasis will be on developing methods for the inverse problem: calculate the required mothership trajectory to achieve a desired drogue trajectory. A Lyapunov-based backstepping algorithm is used to track the mothership trajectory, with the goal of achieving accurate drogue orbits.

II. MATHEMATICAL MODEL OF CABLE-DROGUE SYSTEM

Mathematical models of cable-drogue or towed-cable systems are described in the literature for both air and underwater environments. Most of the methods reported in the literature [2], [3], [8], [9], [10] use techniques that model the cable as a series of $N < \infty$ rigid links with lumped masses at the joints (Figure 2). As recommended in [11], we also followed this approach. However, most researchers develop models based on Euler-Lagrange equations, which do not scale well to a large number of links. As an alternative, we develop the mathematical model of cable-drogue systems using Gauss's Principle, as derived in [12]. A similar approach was used in the context of path planning for UAVs in [13]. Reference [1] gives detailed development of the dynamics of cable-drogue system applying Gauss's principle and the simulation results prove the feasibility of this new approach.

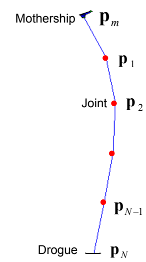

Fig. 2. N-link lumped mass representation of cable-drogue system

III. MOTHERSHIP PATH PLANNING AND CONTROL

A. Mothership orbit calculation using differential flatness

The concept of differential flatness has proved useful in the design of advanced control and supervision schemes for nonlinear systems. Reference [14] introduces flat systems and apply the differential flatness theory to vertical take-off aircraft and articulated ground vehicles. We make use of this property to calculate the inverse dynamics relating a desired drogue orbit to the required orbit of the mothership.

Definition The system $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, t)$ where the states $\mathbf{x} \in \mathbb{R}^n$ and the inputs $\mathbf{u} \in \mathbb{R}^m$, is differentially flat if we can find a set of variables $\mathbf{y} \in \mathbb{R}^m$ called flat outputs and integers r and q such that

$$\begin{aligned} \mathbf{y} &= \mathbf{h}(\mathbf{x}, \mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}}, \cdots, \mathbf{u}^{(r)}) \\ \mathbf{x} &= \mathbf{h}_1(\mathbf{y}, \dot{\mathbf{y}}, \ddot{\mathbf{y}}, \cdots, \mathbf{y}^{(q)}) \\ \mathbf{u} &= \mathbf{h}_2(\mathbf{y}, \dot{\mathbf{y}}, \ddot{\mathbf{y}}, \cdots, \mathbf{y}^{(q+1)}) \end{aligned}$$

that satisfy the system state equations.

Assuming that the only forces on the drogue are aerodynamic forces, gravity, and tension forces from the cable, the cable-drogue system is differentially flat using the trajectory of the drogue as a flat output. Therefore, specifying the desired trajectory of the drogue will dictate the required trajectory for each cable link, and, consequently, for the mothership.

Suppose that the trajectory of the drogue is C^∞. We can then compute the tension components in the N^{th} link of the cable (at the end attached to the drogue) from

$$\begin{aligned} T_N^x &= m_N \ddot{x}_N - F_N^x \\ T_N^y &= m_N \ddot{y}_N - F_N^y \\ T_N^z &= m_N \ddot{z}_N - F_N^z + m_N g, \end{aligned}$$

where F_N^x, F_N^y, F_N^z are the aerodynamic forces acting on the drogue, expressed in the inertial frame. Assuming the length of each link is a constant $l = L/N$, the location of the $(j-1)^{th}$ point mass (located at the $(j-1)^{th}$ joint) is related to the j^{th} point mass using

$$\begin{aligned} x_{j-1} &= x_j + l\frac{T_j^x}{\|\mathbf{T}_j\|} \\ y_{j-1} &= y_j + l\frac{T_j^y}{\|\mathbf{T}_j\|} \end{aligned}$$

$$
\begin{aligned}
z_{j-1} &= z_j + l\frac{T_j^z}{\|\mathbf{T}_j\|} \\
j &= 2,3,\cdots,N,
\end{aligned}
$$

where $\|\cdot\|$ denotes the Euclidean norm. Consequently, the forces on the $(j-1)^{th}$ point mass can be calculated by

$$
\begin{aligned}
T_{j-1}^x &= m_{j-1}\ddot{x}_{j-1} - F_{j-1}^x + T_j^x \\
T_{j-1}^y &= m_{j-1}\ddot{y}_{j-1} - F_{j-1}^y + T_j^y \\
T_{j-1}^z &= m_{j-1}\ddot{z}_{j-1} - F_{j-1}^z + T_j^z + m_{j-1}g \\
j &= 2,3,\cdots,N.
\end{aligned}
$$

At each time step, these equations are applied recursively to each link of the cable until the trajectory of the mothership is calculated.

B. Mothership trajectory tracking using Lyapunov-based control law

The dynamic equations of the mothership can be written as

$$
\begin{aligned}
\dot{p}_n &= V\cos\chi\cos\gamma \\
\dot{p}_e &= V\sin\chi\cos\gamma \\
\dot{p}_d &= -V\sin\gamma \\
\dot{V} &= -g\sin\gamma - \frac{D}{m} + \frac{1}{m}u_T + \frac{F_V}{m} \qquad (1) \\
\dot{\gamma} &= -\frac{g}{V}\cos\gamma\cos\phi + \frac{g}{V}(\cos\phi)u_n + \frac{F_\chi}{mV\cos\gamma} \\
\dot{\chi} &= \frac{L}{mV\cos\gamma}\sin\phi + \frac{F_\gamma}{mV} \\
\dot{\phi} &= u_\phi
\end{aligned}
$$

where $\mathbf{p} \triangleq (p_n, p_e, p_d)^T$ is the current position of the mothership in the inertial frame, V is the airspeed, χ is the heading angle, γ is the flight path angle, ϕ is the roll angle, g is the gravitational constant at Earth sea level, m is the mass of the mothership, D and L are the aerodynamic drag and lift forces respectively, $u_n \triangleq L/mg$ is the (controlled) load factor, u_T is the thrust and (F_V, F_χ, F_γ) is the tension force vector in the velocity coordinates. The control inputs are the thrust u_T, load factor u_n, and roll angle command u_ϕ. The tension forces in the inertial coordinate system can be expressed in the velocity coordinates via the transformation

$$
\begin{pmatrix} F_V \\ F_\chi \\ F_\gamma \end{pmatrix} = \Gamma \begin{pmatrix} T_1^x \\ T_1^y \\ T_1^z \end{pmatrix},
$$

where

$$
\Gamma \triangleq \begin{pmatrix} \cos\gamma\cos\chi & \cos\gamma\sin\chi & \sin\gamma \\ -\sin\chi & \cos\chi & 0 \\ -\sin\gamma\cos\chi & -\sin\gamma\sin\chi & \cos\gamma \end{pmatrix},
$$

and (T_1^x, T_1^y, T_1^z) are the components of tension in the inertial frame for the first cable element connected to the mothership.

Assuming that the desired trajectory $\mathbf{p}^d(t) \in \mathbf{R}^3$ is smooth, and defining the candidate inputs as $\mathbf{u_c} \triangleq (u_T, u_n, \sin\phi)^T$, then rearranging the dynamic equations of the mothership yields

$$
\begin{aligned}
\begin{pmatrix} \dot{V} \\ \dot{\gamma} \\ \dot{\chi} \end{pmatrix} &= \begin{pmatrix} -g\sin\gamma - \frac{D}{m} + \frac{F_v}{m} \\ -\frac{g}{V}\cos\gamma\cos\phi + \frac{F_\chi}{mV\cos\gamma} \\ \frac{F_\gamma}{mV} \end{pmatrix} \\
&\quad + \begin{pmatrix} \frac{1}{m} & 0 & 0 \\ 0 & \frac{g}{V}\cos\phi & 0 \\ 0 & 0 & \frac{L}{mV\cos\gamma} \end{pmatrix} \begin{pmatrix} u_T \\ u_n \\ \sin\phi \end{pmatrix} \\
&= \mathbf{F} + \mathbf{G}\mathbf{u_c},
\end{aligned}
$$

where

$$
\begin{aligned}
\mathbf{F} &\triangleq \begin{pmatrix} -g\sin\gamma - \frac{D}{m} + \frac{F_v}{m} \\ -\frac{g}{V}\cos\gamma\cos\phi + \frac{F_\chi}{mV\cos\gamma} \\ \frac{F_\gamma}{mV} \end{pmatrix}, \\
\mathbf{G} &\triangleq \begin{pmatrix} \frac{1}{m} & 0 & 0 \\ 0 & \frac{g}{V}\cos\phi & 0 \\ 0 & 0 & \frac{L}{mV\cos\gamma} \end{pmatrix}.
\end{aligned}
$$

Theorem 1.1 *Consider the system with the dynamic equations (1) under the stated definitions and assumptions, let*

$$
\begin{aligned}
\xi \triangleq\ & (\mathbf{MG})^{-1} \cdot \Bigg[\frac{d}{dt}\left(\dot{\mathbf{p}}^{\mathbf{d}} - k_1(\mathbf{p} - \mathbf{p}^{\mathbf{d}})\right) - \mathbf{MF} \\
& - (\mathbf{p} - \mathbf{p}^{\mathbf{d}}) - k_2\left(\dot{\mathbf{p}} - \dot{\mathbf{p}}^{\mathbf{d}} + k_1(\mathbf{p} - \mathbf{p}^{\mathbf{d}})\right)\Bigg],
\end{aligned}
$$

where

$$
\mathbf{M} \triangleq \begin{pmatrix} \cos\gamma\cos\chi & -V\sin\gamma\cos\chi & -V\cos\gamma\sin\chi \\ \cos\gamma\sin\chi & -V\sin\gamma\sin\chi & -V\cos\gamma\cos\chi \\ -\sin\gamma & -V\cos\gamma & 0 \end{pmatrix},
$$

and select the control inputs as

$$
\begin{pmatrix} u_T \\ u_n \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}\xi, \qquad (2)
$$

$$
\begin{aligned}
u_\phi = \frac{1}{\cos\phi}\Bigg(& \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}\dot{\xi} \\
& - \left(\dot{\mathbf{p}} - \dot{\mathbf{p}}^{\mathbf{d}} + k_1(\mathbf{p} - \mathbf{p}^{\mathbf{d}})\right)^T \mathbf{MG}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \\
& - k_3\left(\sin\phi - \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}\xi\right)\Bigg). \qquad (3)
\end{aligned}
$$

Suppose that $\mathbf{p}(t) = (p_n, p_e, p_d)^T$ is continuously differentiable. Then, by appropriately selecting the positive constants k_1, k_2 and k_3, there exist control inputs (u_T, u_n, u_ϕ) as given by (2) and (3), such that $\mathbf{p}(t) \to \mathbf{p}^d(t)$ with tracking error $\mathbf{e} \triangleq \mathbf{p} - \mathbf{p}^{\mathbf{d}}$ that is uniformly stable and will exponentially converge to the origin.

Proof:

Step 1. Error dynamics: Let $\mathbf{e} = \mathbf{p} - \mathbf{p^d}$ be the tracking error in the inertial frame. The dynamic equation for the inertial tracking error is then given by

$$\dot{\mathbf{e}} = \dot{\mathbf{p}} - \dot{\mathbf{p}}^{\mathbf{d}}.$$

Step 2. Error convergence: Define the Lyapunov candidate function $V_1 \triangleq \frac{1}{2}\mathbf{e}^T\mathbf{e}$, which has the time derivative

$$\begin{aligned} \dot{V}_1 &= \mathbf{e}^T\dot{\mathbf{e}} \\ &= \mathbf{e}^T(\dot{\mathbf{p}} - \dot{\mathbf{p}}^{\mathbf{d}}). \end{aligned} \tag{4}$$

At this stage of the development, we consider $\dot{\mathbf{p}}$ as a virtual control, where $\dot{V}_1$ can be made negative definite by setting $\dot{\mathbf{p}}$ equal to $\dot{\mathbf{p}}^{\mathbf{d}} - k_1\mathbf{e}$, where k_1 is a positive constant. Introducing the error variable

$$\mathbf{z^d} \triangleq \dot{\mathbf{p}}^{\mathbf{d}} - k_1\mathbf{e},$$

and adding and subtracting $-k_1\mathbf{e}^T\mathbf{e}$ in Equation (4) gives

$$\dot{V}_1 = -k_1\mathbf{e}^T\mathbf{e} + \mathbf{e}^T(\dot{\mathbf{p}} - \mathbf{z^d}).$$

Step 3. Backstepping for $\mathbf{z^d}$: Consider the augmented Lyapunov candidate function

$$V_2 \triangleq V_1 + \frac{1}{2}(\dot{\mathbf{p}} - \mathbf{z^d})^T(\dot{\mathbf{p}} - \mathbf{z^d}),$$

with Lie derivative

$$\dot{V}_2 = -k_1\mathbf{e}^T\mathbf{e} + (\dot{\mathbf{p}} - \mathbf{z^d})^T(\mathbf{e} + \ddot{\mathbf{p}} - \dot{\mathbf{z}}^{\mathbf{d}}).$$

From the mothership dynamic equations, we have

$$\ddot{\mathbf{p}} = \mathbf{M}\,(\mathbf{F} + \mathbf{G}\mathbf{u_c}).$$

Therefore

$$\dot{V}_2 = -k_1\mathbf{e}^T\mathbf{e} + (\dot{\mathbf{p}} - \mathbf{z^d})^T(\mathbf{e} + \mathbf{MF} + \mathbf{MG}\mathbf{u_c} - \dot{\mathbf{z}}^{\mathbf{d}}).$$

It is straightforward to show that $\mathbf{MG}$ is invertible[1], therefore we can pick

$$\xi = (\mathbf{MG})^{-1}\left(\dot{\mathbf{z}}^{\mathbf{d}} - \mathbf{MF} - \mathbf{e} - k_2(\dot{\mathbf{p}} - \mathbf{z^d})\right),$$

where k_2 is a positive constant, and define

$$\eta \triangleq \sin\phi.$$

If we select

$$\begin{pmatrix} u_T \\ u_n \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}\xi, \tag{5}$$

and define

$$z_2^d \triangleq \eta - \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}\xi,$$

the time derivative of the z_2^d can be written as

$$\begin{aligned} \dot{z}_2^d &= \dot{\eta} - \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}\dot{\xi} \\ &= u_\phi\cos\phi - \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}\dot{\xi}, \end{aligned}$$

and

$$\mathbf{u_c} = \begin{pmatrix} u_T \\ u_n \\ \eta \end{pmatrix} = \xi + z_2^d\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

[1]By constraining V, γ, and χ to reasonable values, the matrices $\mathbf{M}$ and $\mathbf{G}$ will be full rank. The product of two full-rank matrices is also full rank.

Thus

$$\begin{aligned} \dot{V}_2 &= -k_1\mathbf{e}^T\mathbf{e} - k_2(\dot{\mathbf{p}} - \mathbf{z^d})^T(\dot{\mathbf{p}} - \mathbf{z^d}) \\ &\quad + (\dot{\mathbf{p}} - \mathbf{z^d})^T\left(\mathbf{MG}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} z_2^d\right). \end{aligned}$$

Step 4. Backstepping for z_2^d: Consider the augmented Lyapunov candidate function

$$V_3 \triangleq V_2 + \frac{1}{2}(z_2^d)^2,$$

with the time derivative

$$\begin{aligned} \dot{V}_3 &= -k_1\mathbf{e}^T\mathbf{e} - k_2(\dot{\mathbf{p}} - \mathbf{z^d})^T(\dot{\mathbf{p}} - \mathbf{z^d}) \\ &\quad + z_2^d\left(\dot{z}_2^d + (\dot{\mathbf{p}} - \mathbf{z^d})^T\mathbf{MG}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right) \\ &= -k_1\mathbf{e}^T\mathbf{e} - k_2(\dot{\mathbf{p}} - \mathbf{z^d})^T(\dot{\mathbf{p}} - \mathbf{z^d}) \\ &\quad + z_2^d\Bigg(u_\phi\cos\phi - \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}\dot{\xi} \\ &\quad + (\dot{\mathbf{p}} - \mathbf{z^d})^T\mathbf{MG}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\Bigg). \end{aligned}$$

If we choose

$$\begin{aligned} u_\phi &= \frac{1}{\cos\phi}\Bigg(\begin{pmatrix} 0 & 0 & 1 \end{pmatrix}\dot{\xi} \\ &\quad - (\dot{\mathbf{p}} - \mathbf{z^d})^T\mathbf{MG}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} - k_3 z_2^d\Bigg), \end{aligned} \tag{6}$$

where k_3 is a positive constant, then the time derivative of V_3 becomes

$$\dot{V}_3 = -k_1\mathbf{e}^T\mathbf{e} - k_2(\dot{\mathbf{p}} - \mathbf{z^d})^T(\dot{\mathbf{p}} - \mathbf{z^d}) - k_3(z_2^d)^2 \le \lambda V_3, \tag{7}$$

where the positive constant λ satisfies

$$0 < \lambda \le 2 \cdot \min\{k_1,\ k_2,\ k_3\}.$$

Therefore, it is straightforward to conclude from the Comparison Lemma [5] that

$$V_3(t) \le e^{-\lambda t}V_3(0),\ t \ge 0.$$

Thus by the appropriate selection of positive constants k_1, k_2, and k_3, the state $(\mathbf{e}^T, (\dot{\mathbf{p}} - \mathbf{z^d})^T, z_2^d)^T$ is uniformly stable and will exponentially converge to the origin. The control inputs (u_T, u_n, u_ϕ) are given by Equation (5) and (6).

□

C. Mothership path planning and control simulation results

In this section, the methods developed in Sections III-A and III-B are used to simulate trajectory calculation and control of the mothership, given a desired drogue orbit. This section aims to validate the proposed trajectory tracking control law, so the simulation will not include wind as a factor, which is a crucial element for aerial vehicles control problem. Future work will include a more exhaustive exploration of the effects of wind.

Based on the configuration of the mothership (a twin prop, 55-inch wingspan, battery-powered, autonomous aircraft with Kestrel autopilot) which is applied in the preliminary flight test, we illustrate several key parameters using in the simulation in Table I. The desired circular trajectory (p_n^{dr}, p_e^{dr}, p_d^{dr}) of the drogue can be written in parametric form as

$$\begin{aligned} p_n^{dr}(t) &= R^{dr}\sin\left(\frac{V^{dr}}{R^{dr}}t\right) \\ p_e^{dr}(t) &= R^{dr}\cos\left(\frac{V^{dr}}{R^{dr}}t\right) \\ p_d^{dr}(t) &= -900 \text{ m}, \\ t &= [0, +\infty). \end{aligned}$$

where R^{dr} and V^{dr} are defined in Table I. The desired initial position $(p_n^m(0), p_e^m(0), p_d^m(0))$, velocity V^m, and radius R^m of mothership to achieve the specified drogue orbit may be calculated using the differential flatness property as

$$\begin{aligned} (p_n^m(0), p_e^m(0), p_d^m(0)) &= (99.57,\ 96.86,\ -908.71) \text{ m} \\ V^m &= 20.84 \text{ m/s} \\ R^m &= 138.91 \text{ m}. \end{aligned}$$

Thus the initial position error of the mothership is $(-99.57,\ 33.14,\ 91.29)$ m.

Figure 3 shows simulation results of the desired (solid) and actual (dashed) trajectories of the mothership, as well as the tracking error in the absence of wind. We see that the mothership tracks the desired trajectory after a 50 second transient. Figure 4 shows simulation results of the desired and actual trajectories of the drogue, as well as the tracking error in the absence of wind. We see that the drogue converges to its desired trajectory after a 70 second transient. Figure 5 shows the time evolution of the characteristic parameters of the mothership. Since the initial position error of the mothership is large compared to the airspeed of the mothership, the control inputs all reach their limits in the first 50 second transient, and after that the mothership enters a steady state. The tension force in the cable acts on the mothership in the centripetal direction. The result is that, even though the roll angle ϕ goes to zero in the steady state, the mothership is still able to fly in a circular orbit.

TABLE I

SIMULATION PARAMETERS

Mothership	Initial Position (0, 130, -1000) m	Airspeed 18-27 m/s	Mass 1.76 kg
Drogue (Desired)	Airspeed $V^{dr} = 15$ m/s	Altitude 900 m	Orbit Radius $R^{dr} = 100$ m
Cable	Mass 0.01 kg	Length 100 m	Diameter 0.01 m
MAV	Airspeed 16.67 m/s		

IV. CONCLUSIONS

In this paper we presented a novel approach to the aerial recovery problem for micro air vehicles. In this approach, a mothership tows a drogue that establishes a stable orbit at a speed that is slow enough to allow the MAV to overtake the drogue as it moves along its orbit trajectory. An inverse dynamics method for calculating the required mothership orbit to achieve a desired drogue orbit was also presented. Using a Lyapunov-based backstepping approach, a control law was designed to enable stable tracking of the required orbit by the mothership.

V. ACKNOWLEDGMENT

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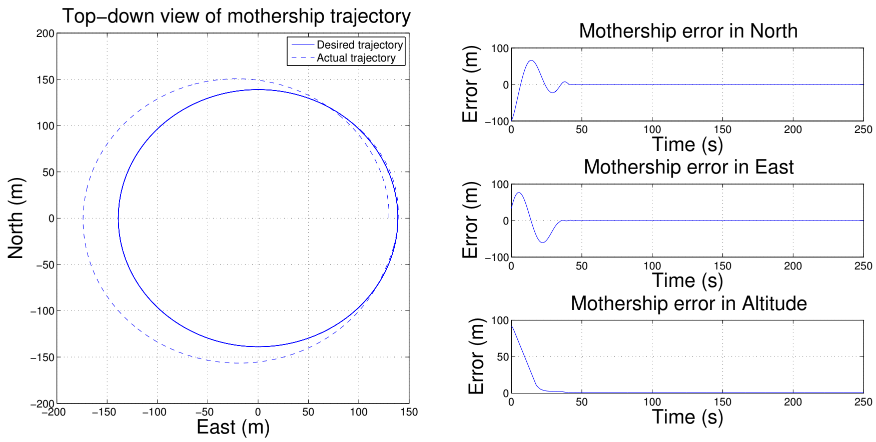

Fig. 3. Mothership trajectory and error driven by Lyapunov-based backstepping control law in the absence of wind.

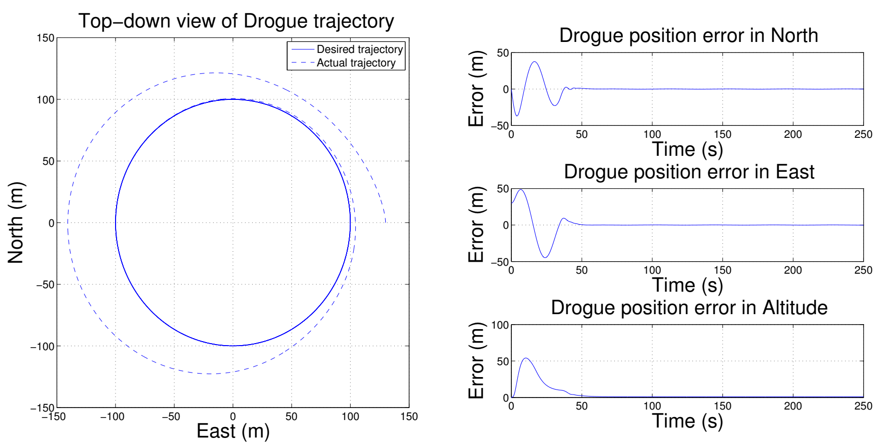

Fig. 4. Drogue trajectory and error driven by Lyapunov-based backstepping control law in the absence of wind

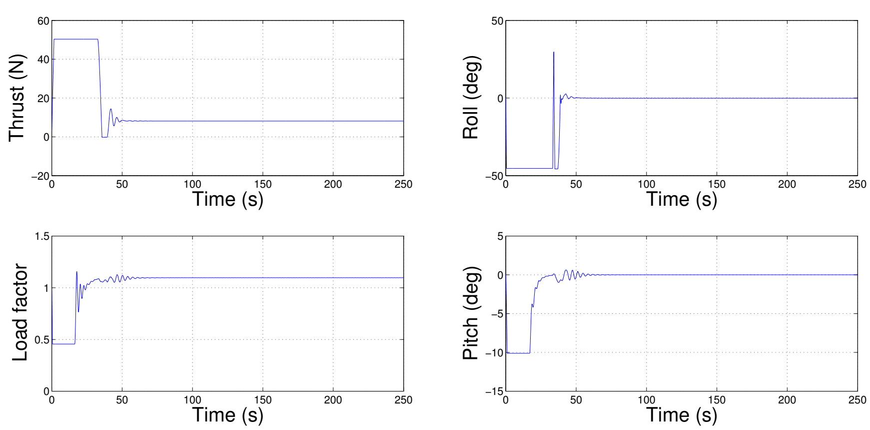

Fig. 5. Time evolution of the thrust T, load factor n, roll angle ϕ, path angle γ of the mothership.